Electoral Alliances and Spatial Patterns of Vote*

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Abstract

This study examines the impact of electoral alliances on candidates' spatial voting patterns, focusing on Brazil's 2020 Municipal Elections. I explore two main hypotheses: the "Spatial Dependence Hypothesis", which posits that electoral coalitions between mayoral and city council candidates increase the spatial dependence of their vote distributions, and the "Brokerage Hypothesis", suggesting that allied council candidates act as local brokers, aligning their vote concentration areas with those of the supported mayoral candidate. Using fixed-effects regression and an instrumental variable approach, the analysis confirms both hypotheses, demonstrating that electoral alliances lead to greater spatial vote dependence and that council candidates significantly influence the supported mayoral candidate's spatial vote concentration. These findings underscore the strategic importance of electoral alliances in potentially enhancing campaign effectiveness through spatial specialization. While the Brazilian context provides a detailed case study, the proposed methodological approach offers broad applicability for analyzing electoral alliances and their effects on spatial vote distributions in various contexts. This research contributes to the literature on electoral alliances, spatial patterns of vote, and electoral strategies, presenting a novel framework for further exploration.

Keywords: Electoral Alliances; Electoral Coalitions; Spatial Patterns of Vote; Electoral Geography; Brazilian Elections

Word Count (excluding appendix): 8322

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1 Introduction

Mobilizing supporters to turn out and persuading swing voters are primary goals of electoral campaigns. While candidates can theoretically seek these goals district-wide, studies show that campaigns are often geographically limited. In particular, candidates tend to receive more votes in their home-places, the "friends and neighbors" voting (e.g. Lewis-Beck and Rice, 1983; Ames, 1995a; Meredith, 2013b). Furthermore, candidates in proportional elections can rely on smaller voter segments to be elected (Cox, 1990; Myerson, 1993), resulting in spatially more concentrated vote distributions compared to majoritarian candidates (e.g. Ames, 1995b; Shugart et al., 2005). Despite the documented rationality behind these spatial voting patterns, there is still a need to understand how electoral cooperation among candidates running for different offices might influence or interact with that individual rationality of spatial specialization.

This paper explores the interaction between candidate spatial voting patterns and electoral coalitions, using Brazil's simultaneous majoritarian and proportional local elections as a case study. Electoral alliances[1], a common phenomenon in executive elections across Europe and South America (Golder, 2006; Kellam, 2017; Spoon and West, 2015; West and Spoon, 2017), involve parties with candidates running for offices in different branches of government forming public alliances for the duration of the election. This research provides a methodological framework for analyzing how such alliances affect candidates' spatial voting patterns, with implications extending beyond the Brazilian context.

By focusing on the polling station level, the most granular geographic electoral unit available, this study shows that executive electoral coalitions lead to greater positive dependence in the spatial voting patterns of majoritarian and proportional candidates. Specifically, it finds that when majoritarian and proportional candidates form alliances, their spatial voting patterns exhibit greater positive dependence. This increase in spatial dependence is primarily due to the spatial concentration areas of supporting council candidates becoming those of the supported mayoral candidate.

Under Brazilian electoral legislation, political parties are allowed to form electoral coalitions within the same constituency (i.e., the same municipality) for mayoral elections, which are held simultaneously with city council elections. Data from 2002-2020 indicate that such coalitions are almost ubiquitous in Brazilian municipalities[2]. However, although electoral alliances in Brazilian elections are commonplace and the literature on the subject is enormous, as pointed out by Limongi and Vasselai (2018), their implications for candidates' mobilization strategies and voting performances remain under-researched.

This paper introduces a novel approach to understanding electoral coalitions' impacts on the spatial patterns of vote distribution, applicable to various electoral contexts. These spatial patterns, which originate from a strand of electoral analysis first suggested by Ames (1995b) and later developed by Avelino et al. (2011) and Silva and Davidian (2013), provide

[1] In this paper, electoral "alliances" and "coalitions" are used interchangeably. Some papers also refer to them as "pre-electoral coalitions".

[2] Complete summary in the appendix (Table B.1).

information on the spatial clustering of candidate support. By examining voting patterns at the polling station level, it is possible to gain a comprehensive understanding of how voter preferences and candidate support vary across different areas, providing valuable insights into electoral dynamics.

To organize the analysis, I outline two hypotheses for empirical testing, relating mayoral electoral coalitions to their consequences on the spatial patterns of vote distributions for mayoral and city council candidates. The first hypothesis, the "Spatial Dependence Hypothesis", posits that electoral alliances between mayoral and city council candidates increase the spatial dependence of their vote distributions. The second hypothesis, the "Brokerage Hypothesis", attributes this increased spatial dependence to council candidates acting as local brokers for the mayoral candidate, with their vote concentration areas becoming those of the supported mayoral candidate.

Proportional elections incentivize council candidates to establish and secure *redutos* ("electoral fortress" or "bailiwicks"), often achieved by building closer relationships with voters and maintaining long-term (and, sometimes, clientelistic) connections, a phenomenon described by Lopez (2004) and Nichter and Peress (2017). As a result, council candidates can serve as valuable local brokers for the electoral interests of mayoral candidates, a notion highlighted by Frey (2024). Thus, mayoral electoral coalitions could function as a cooperation mechanism between mayoral candidates and potential local brokers.

To test these hypotheses rigorously, I employ two main empirical strategies. First, I evaluate the Spatial Dependence Hypothesis using fixed-effects regression, regressing a pairwise measure of spatial vote dependence between mayoral and council candidates on a mayoral electoral alliance indicator while controlling for candidates' and state-party-pair fixed-effects. Second, to assess the Brokerage Hypothesis, I regress the mayoral candidate's measure of spatial vote concentration on the corresponding measure for the allied council candidates, using polling stations' fixed-effects and an instrumental variable approach inspired by Meredith (2013a). This approach uses the locations of allied council candidates' polling places as a source of exogenous variation for their spatial vote concentration.

Additionally, I report a series of robustness checks and extensions, including alternative spatial measures, placebo alliances, and analyses using restricted sets of candidates, to ensure the reliability and validity of the findings. The results strongly support both hypotheses, showing that mayoral electoral coalitions lead to increased spatial vote dependence between mayoral and council candidate pairs and that allied council candidates significantly influence the supported mayoral candidate's spatial vote concentrations.

These findings have important implications for understanding electoral alliances and their consequences for candidates' voting outcomes. Although the Brazilian context provides a detailed case study, the methodological approach presented here can be applied broadly to analyze the effects of electoral alliances or other electoral support agreements on candidates' spatial voting patterns in various contexts. More broadly, this research contributes to at least three strands of the literature. First, it aims to provide a novel approach to the understanding of majoritarian electoral coalitions in Brazilian local elections, relating to an extensive body of literature on electoral alliances in Brazil, starting with Soares (1964)'s seminal strategic interpretation of parties joining electoral alliances to maximize their electoral performances while minimizing efforts. Although much of this literature focuses on alliances in federal and state-level elections (Machado, 2018; Limongi and Vasselai, 2018), electoral coalitions in municipality elections have also been studied, from more empirical (Silva, 2022; Mizuca, 2007) to theoretical approaches (Griebeler and Resende, 2021).

Second, it relates to the literature on the spatial-geographical analysis of elections in Brazil (Avelino et al., 2011, 2016; Silva and Davidian, 2013; Silva and Silotto, 2018; Gelape, 2017), which begins with Ames (1995b)'s seminal work and is further developed by Avelino et al. (2011) and Silva and Davidian (2013). These studies adapt classical concepts and measures of urban economics, such as industrial agglomeration (Ellison and Glaeser, 1997), to the study of spatial patterns of vote distribution. Similarly, this research adapts urban economics concepts and measures, providing a novel interpretation of Ellison et al. (2010)'s industrial coagglomeration index in terms of candidates' spatial patterns of vote distribution dependence. Through these measures and concepts, it is possible to gain new insights into candidates' electoral strategies to geographically limit or expand their campaigns and influence zones.

Finally, since this study hypothesizes and empirically tests that part of the spatial vote distributions of the mayoral candidates is driven by their allied council candidates in the Brazilian municipal elections, it relates to the literature on *coattail effects* (Ferejohn and Calvert, 1984; Rudolph and Leininger, 2021) and political brokerage (Carty, 1981; Gingerich and Medina, 2013). To my knowledge, there are no studies on *coattail effects* in concurrent mayoral and city council elections in Brazil.

The rest of this paper is structured as follows. Section 1 describes the data and measures of spatial vote concentration and dependence; Section 2 outlines the empirical strategy; Section 3 presents the main results; Section 4 discusses heterogeneity analyzes and robustness checks; and Section 5 concludes.

2 Spatial Vote Concentration and Dependence

2.1 Data and Measures

This paper analyzes data from the Brazilian 2020 Municipal Elections. All electoral data was obtained from *Tribunal Superior Eleitoral* (TSE). The main sources are: (i) election data at polling station level[3], from "*voto-seção*" file; (ii) coalition data, from "*coligações*" file; and (iii) candidates' information, from "*candidatos*" and "*filiados*" files. The final data set covers 2020 executive and legislative local elections for all 5,569 Brazilian municipalities.

From electoral data, I first define the S Index, an additive measure of spatial vote concentration at the polling station level in the spirit of Silva and Davidian (2013), who define a similar measure at the municipality level[4]:

[3] In Brazil, a polling station (*local de votação*), is where the electronic ballot boxes are placed, usually a public schools, and where constituents are registered to vote.

[4] In fact, their HC Index is such that $HC_{ilm} = V_{im} * S_{ilm}$.

$$S_{ilm} := \frac{V_{ilm}}{V_{im}} - \frac{V_{lm}}{V_m} \tag{1}$$

where V_{lm} is the turnout at polling station l in municipality m; $V_m := \sum_l V_{lm}$ is the total turnout at municipality m; V_{ilm} is candidate i's number of votes at polling station l in municipality m; and $V_{im} := \sum_l V_{ilm}$ is candidate i's total number of votes at municipality m.[5]

If there is only one polling place in a municipality, the S index is uninformative (i.e., there is no spatial vote variation). Therefore, 213 small municipalities with only one polling place are removed from the population considered in this research. All further analyses are restricted to the remaining 5,356 municipalities. The appendix table B.2 reports the complete distribution of municipalities' number of polling places.

The index S compares a candidate's actual voting performance at a given polling place (that is, $\frac{V_{ilm}}{V_{im}}$) to an expected voting performance equal to the size of the pooling place (that is, $\frac{V_{lm}}{V_m}$), as described by Avelino et al. (2011). If candidate i has a vote concentration in the polling station l (i.e., $S_{ilm} > 0$), then we call l a **dominance area** of candidate i. It is important to note that the S Index does not tell much about the overall voting performance of a candidate at a polling place. As an example, if candidate i has only one vote in the election at l, then $\frac{V_{ilm}}{V_{im}} = 1$ and $S_{ilm} > 0$; which just means that the votes of candidate i' are concentrated at l.

Two alternative measures of spatial vote concentration at the polling station level are defined in appendix A.1: the Horizontal Cluster ($\mathcal{HC}$) and the Locational Quotient ($\mathcal{LQ}$), which were adapted to the context of voting patterns by Silva and Davidian (2013). These alternative measures are further explored in robustness checks.

In order to introduce some stylized facts on the candidates' spatial vote distributions in a municipality, I also define an aggregated index, the G Index, as in Avelino et al. (2011):

$$G_{im} := \sum_l S_{ilm}^2 \tag{2}$$

which measures the dispersion of candidate i's spatial vote distribution across all polling places. The greater the dispersion, the greater candidate's overall vote concentration in the municipality. The G Index has a lower bound at 0, which is reached when a candidate's voting distribution across the polling places (i.e. $\{\frac{V_{ilm}}{V_{im}}\}_l$) is identical to the distribution of polling places' size (i.e. $\{\frac{V_{lm}}{V_m}\}_l$). Again, in appendix A.1, alternative measures of overall vote concentration are defined at the municipality level.

The figures below provide a visual representation of S Index distributions for the four most voted candidates in the 2020 mayoral and council elections in the municipality of Rio de Janeiro. The municipality map is segmented into Voronoi polygons defined from the polling places coordinate set[6]. Thus, each polygon represents a polling place. Blue-shaded polygons

[5] Note that as voters can vote for candidates, parties, or none (null and blank votes), we have that, in general, $V_{lm} \neq \sum_i V_{ilm}$ and $V_m \neq \sum_i V_{im}$.

[6] Coordinates data was obtained from TSE open data.

indicate candidates' dominance areas as measured by the S Index.

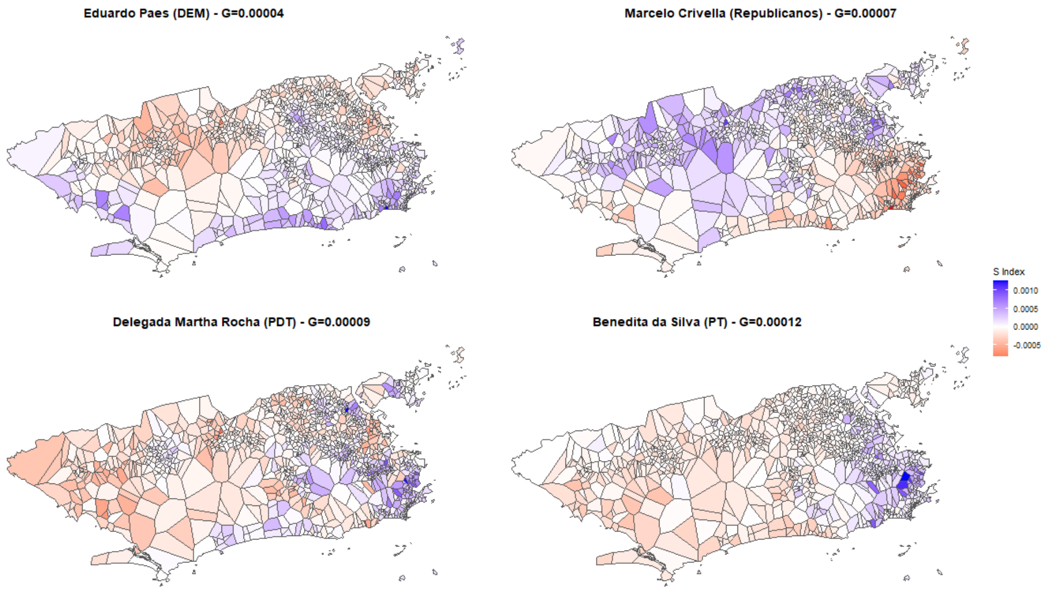

Figure 1: S Index Visual Example - Rio de Janeiro (Mayoral Candidates)

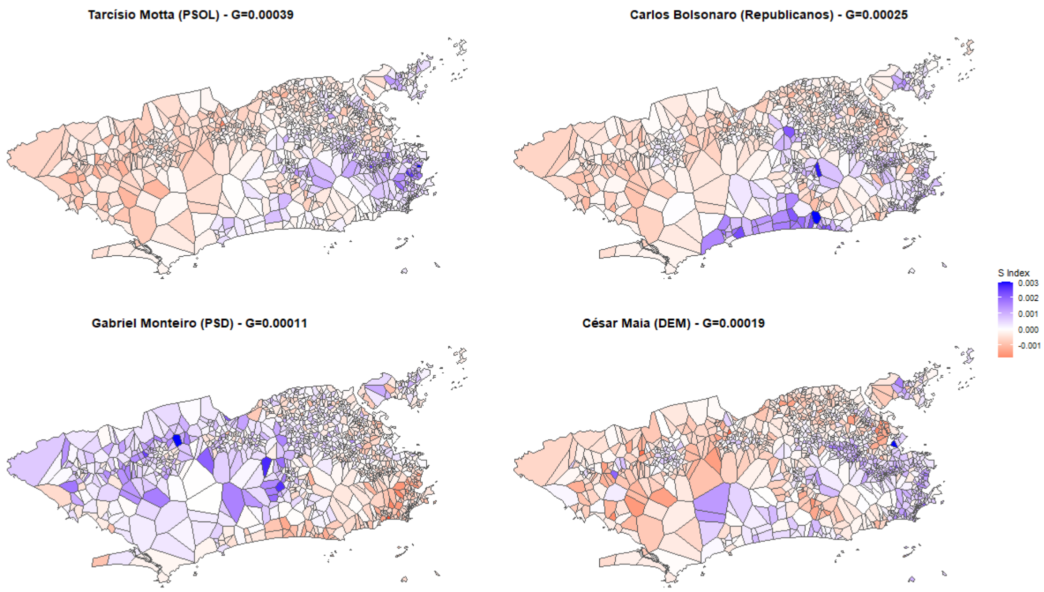

Figure 2: S Index Visual Example - Rio de Janeiro (Council Candidates)

Looking at Rio de Janeiro's mayoral election, Figure 1 indicates, for example, that Benedita da Silva's (PT) dominance areas are concentrated in downtown, while Marcelo Crivella's areas are in the north and west regions. In the council election, Figure 2 indicates that Carlos Bolsonaro's (Republicanos) dominance areas are in the Barra da Tijuca (southwest) region, while Gabriel Monteiro's (PSD) are in the north and west regions. And among the candidates in the two figures, the overall vote concentration in the municipality, given by the G Index, is in general higher for the council candidates.

Finally, to access the impact of mayoral electoral coalitions on executive and legislative candidates' spatial patterns of vote distribution, a measure of pairwise spatial vote concentration dependence is defined. I adapt Ellison et al. (2010)'s industrial coagglomeration index to the context of voting outcomes. Let i be a mayoral candidate and j, a council candidate in municipality m. The spatial vote dependence (SVD) Index for the candidate pair is defined as:

$$SVD_{ijm} := 100 * \frac{\sum_l S_{ilm} * S_{jlm}}{1 - \sum_l (\frac{V_{lm}}{V_m})^2} \tag{3}$$

which is a re-scaled covariance between $\{S_{ilm}\}_l$ and $\{S_{jlm}\}_l$. As pointed out in Ellison et al. (2010)'s mathematical appendix, the correction factor (i.e. $1 - \sum_l (\frac{V_{lm}}{V_m})^2$) eliminate sensitivity to the fineness of the geographic breakdown, as the covariance could be lower due to a more concentrated polling station size distribution. The factor approaches 1 for more scattered distributions of polling places size. A positive SVD Index indicates that two candidates' votes are concentrated in the same polling places, having similar dominance areas, while a negative SVD Index indicates the opposite.

In the appendix section B.3, I present some descriptive tables and charts of the SVD Index distribution for the main studied population and some restricted populations. I also describe an alternative measure of pairwise spatial vote concentration dependence in appendix A.2, the Spatial Adjusted Correlation (SAC), which is the scaled Pearson correlation between $\{S_{ilm}\}_l$ and $\{S_{jlm}\}_l$.

Although in this research I do not develop a theoretical model to rationalize Ellison et al. (2010)'s industrial coagglomeration index in the context of vote outcomes, it may be possible to draw an analogy between the intercity industry location problem (as in Ellison and Glaeser (1997) and O'Sullivan and Strange (2018)) and the candidate problem of deciding where to campaign for votes in an electoral district. Roughly, a candidate could be understood as a firm that maximizes profits (i.e. probability of being elected) and interacts with other firms (i.e. candidates) that have conflicting or mutually beneficial interests, choosing where to open its plants (i.e., where to focus her campaign).

2.2 Empirical Regularities

In this section, some empirical regularities of the candidates' spatial vote distributions in the Brazilian municipal elections for 2020 are introduced. These regularities are relevant to the understanding of mayoral electoral coalitions as a coordination device that links supported mayoral candidate and supporting council candidates' dominance areas.

Regularity 1 *The votes of the city council candidates are spatially more concentrated than the votes of the mayoral candidates.*

The graphs below present the G Index empirical cumulative density functions for council candidates and mayoral candidates. The panel on the left-hand side considers all candidates and on the right-hand side, only effective candidates[7].

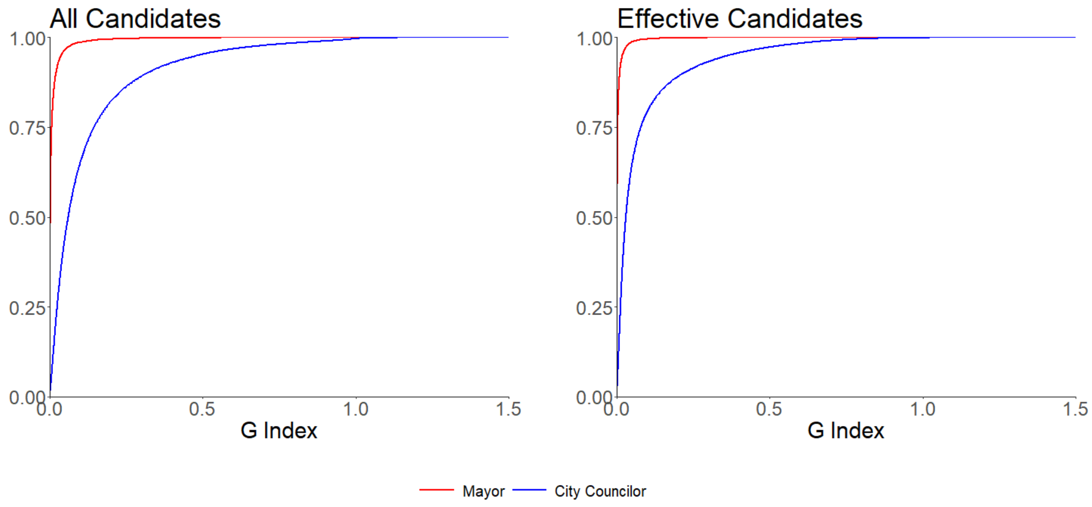

Figure 3: Empirical CDF - G Index

As G Index empirical CDF of council candidates is below mayoral candidates', council candidates' votes are more spatially concentrated than mayoral candidates' votes. The pattern does not change when only effective candidates are considered. As noted in Ames (1995b), because small slices of the electorate can ensure victory in proportional elections, office-seeking candidates in these elections would pursue a voter cohort rather than the median voter (for theoretical reasoning, see, for example, Cox (1990) and Myerson (1993)). Thus, when comparing the spatial patterns of vote distribution between candidates in executive and proportional elections, it would be expected, and the data for 2020 Brazilian local elections confirms, that the latter is more concentrated.

Regularity 2 *The votes of the top-ranked mayoral candidates are more spatially scattered.*

The bar chart below shows that the votes of the higher-ranked mayoral candidates are, on average, more scattered. In other words, the mayoral candidates that receive more votes do so with a more homogeneous voting distribution across the polling places (i.e., more similar to the distribution of polling place size). In the appendix figure B.3, I show that when considering the rank up to the third most voted candidate, conditional on municipalities

[7]That is a restriction to exclude candidates with too few votes. The number of effective candidates in election e in the municipality m follows the classic definition of Laakso and Taagepera (1979):

$$EfCand^e_m := \frac{1}{\sum_i \hat{V}^2_{im}}, \quad e = M, C$$

where $\hat{V}_{im} := \frac{\sum_l V_{ilm}}{\sum_{l,i \in I^e_m} V_{ilm}}$ is the candidate i's share of total nominal votes in the mayoral election, if $e = M$, or in the council election, if $e = C$. I^e_m is the set of candidates participating in election e in the municipality m. Hence, I define candidate i as an *effective candidate* in municipality m election e if:

$$rank_m(i) \leq \lceil EfCand^e_m \rceil$$

where $rank_m(i) := \#\{s | V_{im} < V_{sm}\} + 1$.

where three or more mayoral candidates have competed, there is an even sharper difference between the mean concentration of the second and the third most voted candidates.

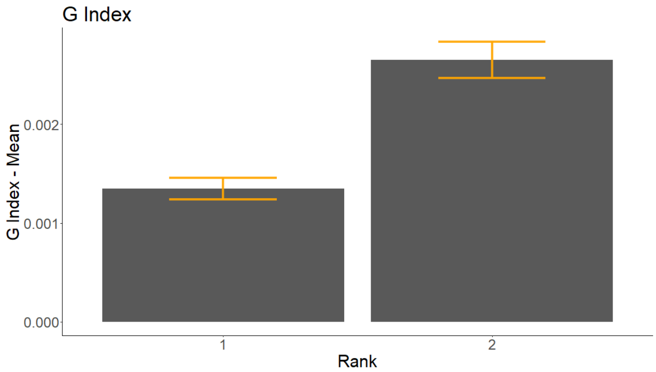

Figure 4: Mean Concentration by Voting Rank - Mayoral Elections

Regularity 3 *The votes of the top-ranked council candidates are more spatially scattered, but still more concentrated than the votes of the top-ranked mayoral candidates.*

Finally, the bar chart below shows the third empirical regularity in the spatial vote distributions of the 2020 local elections. As well as for mayoral candidates, most voted council candidates' votes are, on average, more spatially scattered. However, as expected from the first regularity, these votes are still much more concentrated than those of mayoral candidates - which becomes clear by looking at the different scales on the y-axis in figures 4 and 5. Whereas the mean G index of the most voted mayoral candidates is about 0.0015, for the council counterpart the mean G Index is almost 33 times larger (circa 0.05).

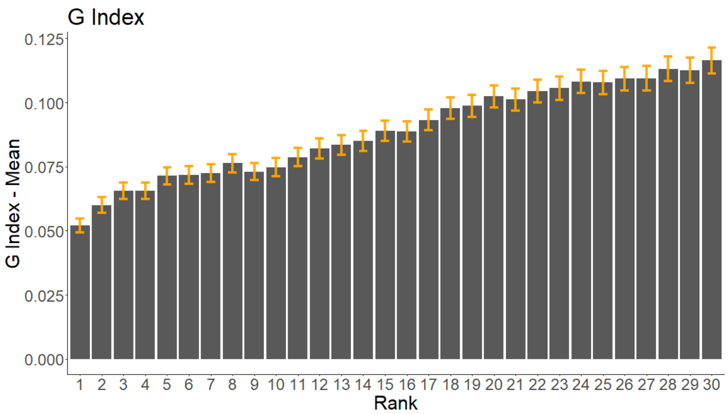

Figure 5: Mean Concentration by Voting Rank - Council Elections

As a robustness exercise, in appendix B.2 the above graphs are reproduced considering alternative measures of aggregated spatial vote concentration, the S_{Max} and the C Index. There is no change in the empirical regularities discussed.

2.3 Main Hypotheses

What do these empirical regularities tell us about electoral alliances? First, to win a majoritarian election, the mayoral candidate cannot rely on voters from a specific area only, as indicated by the second empirical regularity. Therefore, a mayoral candidate should try to maximize her votes in different areas across the municipality. But campaigning in different areas, convincing different groups of voters to vote for her, must be costly. Each group has specific needs, which might be difficult to determine if there is no previous or constant relationship. And that is where electoral alliances enter.

The empirical regularities presented above indicate that city council candidates, on the other hand, can rely on voters from a more spatially restricted area and secure victory. As described by Lopez (2004) and Nichter (2018), city council candidates are typically closer to voters and often secure their dominance areas (i.e., their *redutos*) with long-term (and, sometimes, clientelistic) relationships. A mayoral electoral coalition would thus be a way to connect the mayoral candidate to council candidates such that the latter's dominance areas, their local cluster of voters, also vote for the former, i.e. the allied mayoral candidate. In other words, council candidates would act as local brokers for their allied mayoral candidates.

That council candidates can be useful local brokers for mayoral candidates was already pointed out by Frey (2024). However, the role of electoral alliances in this brokerage relationship has not yet been empirically tested. Thus, the first hypothesis to be empirically tested is an equilibrium result, the "Spatial Dependence Hypothesis", which establishes how mayoral coalitions affect the relation between mayoral and council candidates' spatial vote distributions:

Hypothesis 1 *The spatial vote distributions of the mayoral and city council candidates become more positively dependent when their parties are allied in a mayoral electoral coalition.*

In other words, when mayoral and council candidates' parties are allied in a mayoral electoral coalition, we expect their dominance areas to be more similar than if they were not allied. But what is behind this increased dependence? My second hypothesis, the "Brokerage Hypothesis", tries to unveil the "mechanism" that explains the first hypothesis' equilibrium result.

Hypothesis 2 *Council candidates act as local brokers for the mayoral candidate in their mayoral electoral coalition, such that the vote concentration areas of the former turn into vote concentration areas of the latter.*

Thus, when mayoral and council candidates' parties are allied in a mayoral electoral coalition, we expect their dominance areas to be more similar than if they were not allied because council candidates act as local brokers for the allied mayoral candidate and the former's dominance areas become also the latter's dominance areas.

3 Empirical Strategy

In this section, I describe and discuss empirical strategies for testing the two main hypotheses in this research.

3.1 Spatial Dependence Hypothesis

In order to estimate the impact of being in the same mayoral electoral coalition on the spatial dependence of the pair of mayoral and council candidates, I follow a fixed-effects specification, similar to Steijn et al. (2022). Let i be a mayoral candidate and j be a council candidate, both from the municipality m of the state s. The main fixed-effects specification is given by:

$$SVD_{ijms} = \beta Allied_{ijms} + \mu_{ims} + \gamma_{jms} + \omega_{ijs} + \epsilon_{ijms} \tag{4}$$

where μ_{ims} and γ_{jms} are individual candidates' fixed-effects, and ω_{ijs} is i and j's pair of parties fixed-effect in state s. $Allied_{ijms}$ is an indicator variable equal to 1 if i and j's parties are allied in i's mayoral electoral coalition, and to 0 otherwise. ϵ_{ijms} is the idiosyncratic error term. The parameter of interest, β, is the mayoral electoral coalition average effect on the mayoral and council candidates pair's spatial vote dependence, measured by the SVD Index. If it is correctly identified and the Spatial Dependence Hypothesis is true, we must find β to be positive and statistically different from zero.

It is important to stress which pairs of candidates for mayor and city council are considered in the estimation of Equation 4. As candidates from the same party are always allied, the estimation considers only council candidates from parties that do not launch a mayoral candidate in the municipality. That restriction is relevant because, taking mayoral candidacies as given, there is no counterfactual in which council candidates from a party that launches a mayoral candidate are not in that candidate's mayoral electoral coalition.

Finally, since there is no previous work that explores Ellison et al. (2010)'s industrial coagglomeration index in the context of voting outcomes, it might be difficult to interpret the magnitude of the estimated effect. To provide a baseline comparison, I consider all the pairs of mayor and city council candidates and estimate the following regression:

$$SVD_{ijms} = \bar{\beta} SameParty_{ijms} + \bar{\gamma}_{jms} + \bar{\omega}_{ijs} + \bar{\epsilon}_{ijms} \tag{5}$$

where $SameParty_{ijms}$ is an indicator variable equals to 1 if i and j's parties are the same, and to 0 otherwise. As before $\bar{\mu}_{ims}$ and $\bar{\gamma}_{jms}$ are individual candidates' fixed-effects. The parameter $\bar{\beta}$, the average effect on the SVD Index of mayoral and council candidates' parties being the same, can be used as a baseline comparison to β. If the dependence of candidates' spatial vote concentration distributions is greater when both are from the same party, as descriptive statistics in the appendix indicate, then showing how close the increase in this dependence due to being in the same electoral coalition is to the increase due to being from the same party can shed light on the relevance of the estimated effect of electoral alliances (i.e. β).

3.2 Brokerage Hypothesis

To test the second hypothesis, it is necessary to assess the impact of allied council candidates' dominance areas on the mayoral candidate's dominance areas. If the Brokerage Hypothesis is true, then we expect the impact to be positive and statistically different from zero. To test this, I consider only the population of mayoral candidates supported by a mayoral electoral coalition and follow a fixed-effects specification:

$$S_{ilm} = \delta S_{ilm}^{Allied} + \alpha_{lm} + \epsilon_{ilm} \tag{6}$$

where S_{ilm} is mayoral candidate i's measure of spatial vote concentration at polling place l in municipality m, S_{ilm}^{Allied} is the same measure for the council candidates in i's mayoral electoral coalition, excluding candidates from the same i' party, and α_{lm} is the polling place fixed-effect. ϵ_{ilm} is the idiosyncratic error term. The parameter of interest is δ, which is an average effect of a marginal change in S_{ilm}^{Allied} on S_{ilm}. If it is correctly identified and the Brokerage Hypothesis holds, this effect should be positive and statistically different from zero.

Attention must be drawn to why following a specification as that of Equation 6. As voters can vote for one and only council candidate, a change in council candidates in i's mayoral electoral coalition dominance areas must also change the dominance areas of other council candidates', party voting or blank and null voting, which, in turn, also affect mayoral candidate i's dominance areas. Hence, by not controlling for these other dominance areas of city council elections in the main specification, δ must be understood as an average **net** effect of a marginal change in S_{ilm}^{Allied} on S_{ilm}.

As there are no control variables for mayoral candidacies at the polling station level (e.g., their campaign effort at each polling station), an omitted variable bias is still an identification concern in the specification of Equation 6. Surely there are interactions between mayoral candidates' and voters' characteristics at the polling station level that affect both her dominance areas and that of her council candidates' allies, such as ideological affinity. Moreover, there may also be a reverse causality concern. Not only do allied council candidates' dominance areas affect mayoral candidates' dominance areas, but also mayoral candidates' dominance areas could affect the dominance areas of their allied council candidates.

To address these potential concerns, I follow an instrumental variable approach. In this approach, I will use a *Friends-and-Neighbors*-like instrument, as described in (Meredith, 2013a), to instrument allied council candidates' dominance areas. This instrument exploits votes centered on a candidate's local ties and personal connections, which are considered a form of personal vote (Fiva and Smith, 2017). The instrument Z_{ilm} is defined as the proportion of i' allied council candidates who vote at the polling station l.[8]

The validity of this instrumental approach relies on two main restrictions. The first,

[8]The data used to assess at which polling place each candidate vote was last updated a year before the 2020 elections. So, there are candidates who had not yet joined a political party and it is not possible to assess these candidates' polling places. The instrument must be taken as a lower bound for the actual proportion of candidates who vote at the polling place.

the *relevance restriction*, is expected to hold, as candidates vote at polling stations close to where they live or grew up, which turns out to be the areas where they are typically closer to voters. So, the greater the proportion of allied council candidates voting in a polling place, the greater their voting concentration at this polling place is expected to be.

For the second one, the *exclusion restriction*, there must be the case that the proportion of allied council candidates voting at a polling station affects the mayoral candidate's dominance areas only through the impact on the allied council candidates' dominance areas. As where candidates vote is, in general, defined at the moment of the first voter registration, which is mandatory in Brazil for individuals aged 18 and over, there is not much room for changes depending on the configuration of the elections. Thus, it seems plausible that where allied council candidates vote only affects the mayoral candidate's dominance areas if affecting these council candidates' own dominance areas.

One interesting aspect of Equation 6's specification and the proposed instrumental variable approach is that they can be easily adapted to estimate the other way round: the effect of mayoral candidate's dominance areas on their allies council candidates' dominance areas. Theoretically, there is no reason to expect the effect to be one-sided only. As demonstrated by Zudenkova (2011) in a political agency model of coattail voting, a context similar to that considered in this paper, "two-sided" coattail effects are a possible observable outcome. For example, Garmendia Madariaga and Ozen (2015) find a reciprocal relationship between presidential and gubernatorial vote shares at the state level in US elections. Considering the *Friends-and-Neighbors* variation, there might be both voters that become a mayoral candidate's voters because of the personal vote in a supporting council candidate and voters that become a council candidate's voters because of the personal vote in the supported mayoral candidate.

The "inverse" specification is given by:

$$S_{ilm}^{Allied} = \breve{\delta} S_{ilm} + \breve{\alpha}_{lm} + \breve{\epsilon}_{ilm} \tag{7}$$

and the inverse instrumental variable for S_{ilm}, $\breve{Z}_{ilm}$, is an indicator variable equal to 1 if mayoral candidate i votes at polling place l.

Finally, similarly to the discussion of the Spatial Dependence Hypothesis' empirical strategy, it is important to consider a baseline comparison for the estimated parameter of interest. Following that reasoning, I consider the same population of candidates from the estimation of Equation 6 (i.e., mayoral candidates supported by a mayoral electoral coalition) and estimate:

$$S_{ilm} = \bar{\delta} S_{ilm}^{SameParty} + \bar{\alpha}_{lm} + \bar{\epsilon}_{ilm} \tag{8}$$

where $S_{ilm}^{SameParty}$ is the spatial vote concentration measure at polling place l of council candidates from the same mayoral candidate i's party. The baseline comparison parameter, $\bar{\delta}$, is thus the average **net** effect of a marginal change in $S_{ilm}^{SameParty}$ on S_{ilm}. The dominance areas of same party city council candidates, $S_{ilm}^{SameParty}$, is also instrumented by $\bar{Z}_{ilm}$, which is defined as the proportion of same party council candidates that vote at polling place l.

4 Main Results

4.1 Spatial Dependence Hypothesis

In table 1, I present the main results following the empirical strategy detailed in section 3.1 for the Spatial Dependence Hypothesis' test. In the first column, the estimated model consists of a simple OLS regression of the SVD Index on the mayoral electoral alliance indicator. Columns 2 to 4 add progressively fixed effects of the state party pair, the mayoral candidate, and the city council candidate. The model in column 4 is the main fixed-effects specification described in Equation 4. All standard errors are two-way clustered at the mayoral candidate and council candidate levels.

Dependent Variable:	SVD Index			
	OLS	FE		
Model:	(1)	(2)	(3)	(4)
Variables				
Allied	0.0759***	0.0974***	0.1346***	0.1444***
	(0.0051)	(0.0062)	(0.0066)	(0.0073)
Fixed-effects				
Party Pair & State		Yes		Yes
Mayoral Cand. i			Yes	Yes
City Council Cand. j			Yes	Yes
Fit statistics				
Observations	1,424,901	1,424,901	1,424,901	1,424,901

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 1: Spatial Dependence Hypothesis' Test

The estimates indicate that being in the same mayoral electoral coalition indeed increases mayoral and council candidate pair's spatial vote dependence. In other words, mayoral and council candidates' dominance areas become more similar when the two are allied in a mayoral electoral coalition, as stated in the Spatial Dependence Hypothesis. The complete fixed-effects model point estimate, 0.14, is positive and statistically significant at the 1% level. Considering the descriptive statistics of the sample SVD index in the appendix table B.4, the estimated effect corresponds to almost a one-tenth standard deviation.

The appendix table C.1 reports Equation 5's estimate considering all pairs of mayoral and council candidates, including those with candidates from the same party. Considering the main estimate in table 1, 0.1444, the effect of mayoral electoral coalition on the candidate pair SVD is about 28% of the same party effect, 0.5095. Thus, being in the same mayoral electoral coalition does not increase the candidate pair's dominance areas dependence as much as being in the same party, but still represents a non-negligible effect.

4.2 Brokerage Hypothesis

Now turning to the Brokerage Hypothesis, table 2 below reports Equation 6 estimates considering three alternative measures of candidates' spatial vote concentration at polling stations: the S Index, the Horizontal Clustering (HC) and the Locational Quotient (LQ)[9]. Odd columns consider a fixed-effects specification, while even columns report 2SLS estimates instrumenting allied council candidates' spatial vote concentration with the proportion of these allied council candidates that vote at the particular polling station. Standard errors are clustered at the municipality level.

The instrumental variable model's estimates confirm that a marginal increase in allied council candidates' spatial vote concentration increases the supported mayoral candidate's spatial vote concentration, as expected from the Brokerage Hypothesis. This positive and statistically significant effect is robust to the different measures of spatial vote concentration. Table C.14 in the appendix reports the first-stage results for even columns' 2SLS estimates, which show that the instrument is significantly correlated with allied council candidates' dominance areas in all three measures of spatial vote concentration.

Dependent Variables:	M Cand. S		M Cand. HC		M Cand. LQ	
	OLS	2SLS	OLS	2SLS	OLS	2SLS
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
Allied CC Cand. S	0.1477***	0.1463***				
	(0.0077)	(0.0098)				
Allied CC Cand. HC			0.2682***	0.4730***		
			(0.0319)	(0.0205)		
Allied CC Cand. LQ					0.0903***	0.1388***
					(0.0220)	(0.0070)
Fixed-effects						
Polling Place	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	247,792	247,792	247,792	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 2: Brokerage Hypothesis' Test

Considering the S Index, a 1 p.p. increase in allied council candidates' spatial vote

[9]The Horizontal Clustering (HC) and the Locational Quotient (LQ) are defined by equations A.9 and A.10 in the Appendix.

concentration due to the *Friends-and-Neighbors* variation causes a 0.15 p.p. net increase in the mayoral candidate's spatial vote concentration. The Horizontal Clustering (HC) measure translates the S Index in terms of candidates' votes. Thus, a 1 vote marginal increase in allied council candidates' spatial vote concentration causes a 0.47 net vote increase in the mayoral candidate's spatial vote concentration.

Table 3 reports the estimates of Equation 7, the "inverse" specification, again considering the three different measures of candidates. Odd columns consider a fixed-effects specification, while even columns report 2SLS estimates instrumenting the mayoral candidate's spatial vote concentration with an indicator of whether she votes at the particular polling station. First-stage estimates are shown in the appendix table C.15.

Dependent Variables:	Allied CC Cand. S		Allied CC Cand. HC		Allied CC Cand. LQ	
	OLS	2SLS	OLS	2SLS	OLS	2SLS
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
M Cand. S	0.5211***	0.1839***				
	(0.0230)	(0.0625)				
M Cand. HC			0.1897***	0.1194***		
			(0.0258)	(0.0207)		
M Cand. LQ					0.3219***	0.2193***
					(0.0333)	(0.0428)
Fixed-effects						
Polling Place	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	247,792	247,792	247,792	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 3: Brokerage Hypothesis' Test - "Inverse"

Point estimates in the instrumental variable models for all three measures of spatial vote concentration are positive and statistically significant, indicating that there is also a "inverse" effect: a marginal increase in the mayoral candidate's spatial vote concentration also increases allied council candidates' spatial vote concentration.

To compare the estimates from tables 2 and 3, it is important to consider that council candidates' votes are much more spatially concentrated than mayoral candidates' votes, as discussed in section 2.3. Thus, when increasing mayoral and council candidates' vote concentration by the same amount, the relative increase must be higher for the former. This argument becomes more clear in the appendix's tables C.17 and C.18, which report tables 2 and 3's estimates considering the measures of spatial vote concentration after standardization. Comparing the estimates from both tables, it is clear that the brokerage effect of allied council candidates' vote concentration is stronger than the "inverse" effect of mayoral candidate's vote concentration (by more than three times). Therefore, the evidence suggests that what drives the increase in the dependence of mayoral and allied council candidates' dominance areas is indeed council candidates acting as local brokers for the mayoral candidate, as expected from the Brokerage Hypothesis.

Finally, the appendix table C.13 reports Equation 8's estimates, considering the regression of the mayoral candidate's spatial vote concentration on the spatial vote concentration of council candidates from her same party, for the three different measures of spatial vote concentration[10]. Compared to the 2SLS estimates in Table 2, the impact of the dominance areas of the same-party city council candidates on the dominance areas of the mayoral candidate is greater for all three measures of spatial vote concentration. For the S Index, the net impact of allied council candidates is about 54% of the impact of same-party council candidates. Thus, the results indicate that council candidates allied to a mayoral candidate in a mayoral electoral coalition can be as useful brokers to the supported mayoral candidate as same-party council candidates.

5 Extensions

In this section, I discuss some extensions and robustness checks to the main empirical specifications discussed in section 3. First, subsection 5.1 reports the extensions' exercises for the Spatial Dependence Hypothesis test. Subsection 5.2 reports the analogous and additional exercises for the Brokerage Hypothesis test.

5.1 Spatial Dependence Hypothesis

5.1.A Alternative Measures

As mentioned in section 2.1, to assess the impact of mayoral electoral coalitions on mayoral and council candidates' patterns of vote distribution, I consider yet an alternative measure to the SVD Index, the Spatial Adjusted Correlation (SAC), which is defined as the scaled Pearson correlation between the paired candidates' spatial vote distributions (see Equation A.13 in the appendix). Thus, to test the Spatial Dependence Hypothesis, I replicate the empirical strategy described in section 3.1, but considering the SAC as the dependent variable.

Table C.2 in the appendix reports the main results analogously to table 1. The estimates again confirm that being in the same mayoral electoral coalition increases mayoral and council candidate pairs' spatial vote dependence. The complete fixed-effects point estimate, 3.537, is positive and statistically significant at the 1% level. This effect represents approximately 7% of the sample mean, as the Spatial Adjusted Correlation mean is close to 50 by design.

5.1.B IV Analysis

In Section 3.1 it was mentioned that the main concern in estimating the specification outlined in Equation 4 is an omitted variable bias. There may be some party-pair variable at

[10]First-stage estimates are shown in appendix table C.16

the municipality level that affects both the SVD Index and the mayoral electoral coalition indicator and is not controlled for in the proposed specification. To mitigate this issue, I propose instrumenting the mayoral electoral coalition indicator, $Allied_{ijms}$ with a *leave-one-out* instrument, defined in appendix equation C.14, which considers variation the mean tendency of parties alliances in farther municipalities as source of exogenous. The approach is further discussed in the appendix section C.1.C.

The appendix table C.3 reports the main fixed-effects estimate (from Equation 4) in column 1 and the 2SLS estimate in column 2. The point estimate, 0.17, is again positive and statistically significant at the 1% significance level. It is close to the complete fixed-effects point estimate, 0.14, supporting the validity of the Spatial Dependence Hypothesis. In appendix section C.1.D, I also propose a robustness check to the *leave-one-out* instrument, considering alternative definitions of the instrument. The exercise reinforces the validity of the Spatial Dependence Hypothesis.

5.1.C Heterogeneity

In the appendix, I report heterogeneity analyses in three dimensions that might impact the effect of mayoral electoral coalitions on candidates' spatial vote dependence: municipality size, mayoral candidate incumbency status, and mayoral candidate party. For each analysis, I consider a fixed-effects specification, as that of Equation 4, but interacting the mayoral electoral alliance indicator with categorical indicators representing the dimension of heterogeneity.

First, table C.7 reports the heterogeneity analysis of the size of the municipality. The mayoral electoral alliance indicator is interacted with three indicators, each indicating if the municipality's number of polling stations is between the first and second quartiles ($Q2\_PS$), between the second and the third quartiles ($Q3\_PS$) and above the third quartile ($Q4\_PS$). The estimates, both considering the SVD Index and the SAC as dependent variables, indicate that the mayoral electoral coalition's positive effect on mayoral and council candidate pairs' spatial vote dependence decreases with the number of polling stations in a municipality.

Second, table C.8 reports the heterogeneity in the mayoral incumbency status. The mayoral electoral alliance indicator is interacted with an indicator that is equal to 1 when the mayoral candidate is the incumbent mayor or, if no candidate is the incumbent mayor, when the mayoral candidate's party is the incumbent mayoral party, and equal to 0 otherwise. The estimates indicate that the mayoral electoral coalition's positive effect on mayoral and council candidate pairs' spatial vote dependence decreases when the mayoral candidacy is the incumbent candidacy. However, the evidence is not robust to both the SVD Index and the SAC as dependent variables. The estimate is not statistically significant when considering the latter.

Finally, table C.9 reports the heterogeneity analysis considering the different mayoral parties. The mayoral electoral alliance indicator is interacted with five indicators, each indicating if the mayoral candidate is from one of the five parties that launched the most mayoral candidates in 2020 municipality elections; i.e. MDB, PSD, PP, PSDB, and PT. The estimates indicate that evidence of heterogeneity in this dimension is weak. Overall, the point estimates are not statistically significant and are not robust to both the SVD Index and the SAC as dependent variables.

5.1.D Party Aggregation and Effective Candidates

A possible problem related to the empirical strategy to test the Spatial Dependence Hypothesis, described in section 3.1, is that there might be mayoral and council candidates that are not competitive and get few votes. As suggested in section 2.1, the main spatial concentration measure, the S Index, which is the building block for both the SVD index and the SAC, can be distorted, assuming extreme values, when a candidate has few votes. Thus, I propose two alternative specifications to prevent this "weak-candidates" potential bias.

First, table 1's models are reestimated considering the pairs of mayoral candidates and council candidates aggregated by party [11]. In the appendix, table C.10 reports the exercise. Again, the estimates confirm that being in the same mayoral electoral coalition increases mayoral and council candidate pairs' spatial vote dependence. The complete fixed-effects estimate, 0.25, is greater than the point estimate in table 1, 0.14.

Secondly, table 1's models are reestimated considering only the pairs of effective mayoral candidates and effective council candidates, as defined in footnote 7. Once more, the estimates, reported in table C.11, confirm that being in the same mayoral electoral coalition increases the spatial vote dependence of the pairs of mayoral and council candidates. The complete fixed-effects point estimate, 0.24, is greater than the main specification point estimate, 0.14. The effect of the mayoral electoral coalition on the SVD of the pair is about 48% of the same party effect, which is greater than the relative effect considering mayoral and council candidate pairs.

Overall, both exercises go hand in hand with the main specification's results reported in table 1. The estimates support the Spatial Dependence Hypothesis.

5.1.E Donations

Could the estimated mayoral electoral coalition effect on mayoral and council candidates' spatial vote dependence be explained by another relation between allied mayoral and council candidates? As mayoral intercandidate campaign donations are only allowed when both candidates are in the same party or allied in the same mayoral electoral coalition, the mayoral electoral coalition effect could in fact be explained by this financial relationship. Put differently, it could be the case that what strengths mayoral and council candidates' spatial vote dependence is not being allied in a mayoral electoral coalition, but rather the inter-candidates' donations inside the coalition. To investigate it, I expand the specification of equation 4 including an indicator variable equal to 1 when the mayoral candidate made a campaign donation to the council candidate in the pair, and to 0 otherwise.

[11] And as before, the estimation considers only council parties that do not launch a mayoral candidate in the municipality

Table C.12 in the appendix reports the results of the expanded specification. Considering both the complete fixed-effects model (column 4) and the 2SLS model (column 5), considering the instrumental variable approach discussed in section 5.1.B, the inclusion of the campaign donation control does not change much the mayoral electoral coalition effect on mayoral and council candidates' spatial vote dependence, which indicates that it is not the financial relationship in the mayoral electoral alliance that explains the effect.

5.1.F Placebo Coalitions

As a robustness check, I perform a placebo treatment exercise in the spirit of a permutation test. Taking mayoral candidacies in a municipality election as given, to construct a placebo mayoral electoral coalition, I randomly select for each party participating in the city council election a mayoral candidacy to support, including the possibility of supporting no candidacy. I consider 250 repetitions of the exercise and, for each set of placebo mayoral electoral coalition, I estimate the models in table 1's column 4 and table C.3's column 2 (i.e. the complete fixed-effects model and the 2SLS model). In the figure below I present the distribution of mayoral electoral placebo coalitions' estimated effect on the SVD Index. The estimated effects present in tables 1 and C.3 are shown by a vertical dotted line.

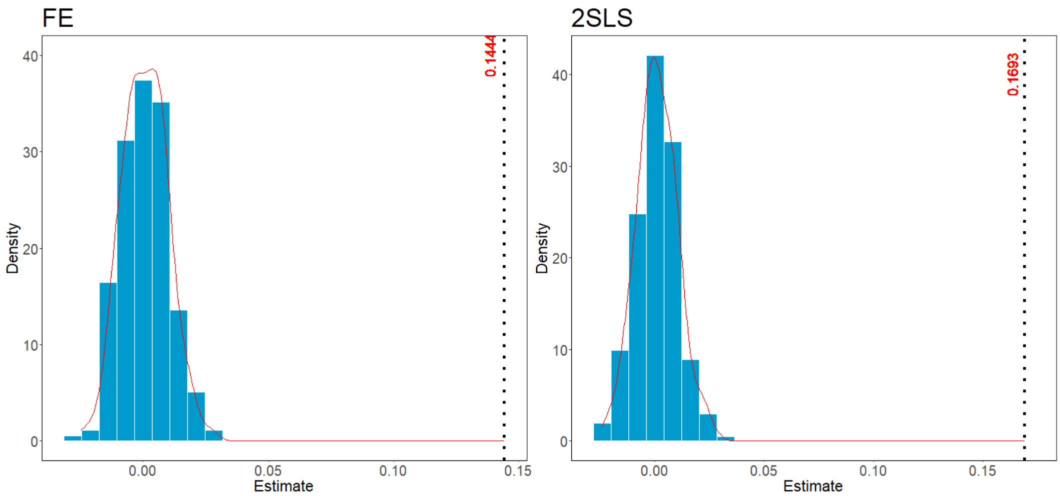

Figure 6: Placebo Coalitions - Estimates Histograms

The mayoral electoral coalitions' estimated effects on the mayoral and council candidates pair's spatial vote dependence, both in the complete fixed-effects and in the IV specifications, are outliers in the distributions of placebo mayoral electoral coalitions' estimated effects. Thus, the exercise reinforces that mayoral and council candidates' dominance areas indeed become more similar when they are allied in a mayoral electoral coalition, as stated in the Spatial Dependence Hypothesis.

5.2 Brokerage Hypothesis

5.2.A Alternative Measures

As with the Spatial Dependence Hypothesis' test, I also consider alternative measures of spatial vote concentration to the S Index when testing the Brokerage Hypothesis. The two alternative measures, Horizontal Clustering (HQ) and Locational Quotient (LQ), are defined by equations A.9 and A.10 in the appendix, following Silva and Davidian (2013). The estimates considering these alternative measures are present simultaneously with the S Index' estimates, in section 4.2's results tables. Overall, the estimates considering alternative measures of spatial vote concentration reported in table 2 (columns 3 to 6) confirm that a marginal increase in allied council candidates' spatial vote concentration increases the supported mayoral candidate's spatial vote concentration, as predicted by the Brokerage Hypothesis.

5.2.B Coattail Effects Framework

Zudenkova (2011) defines the coattail effect as "*the tendency of a popular candidate for one level of government to attract votes to candidates from the same political party for other levels of government*". In this paper, I focus on spatial patterns of vote distributions, which provide information on the clustering of candidates' support, rather than raw vote shares, distinguishing it from most coattail effects literature (e.g. Ferejohn and Calvert, 1984; Ames, 1994; Samuels, 2000; Meredith, 2013a).

However, the specifications in equations 6 and 7 can be easily adapted to a more traditional "coattail effects framework" by replacing the spatial vote concentration measures with the vote shares at the polling station level (that is, $\frac{V_{ilm}}{V_{lm}}$). Besides, the IV analysis should remain the same, instrumenting candidates' vote shares with the *friends-and-neighbors* instrument as defined in section 3.2 - which closely aligns with the original use of the instrument in Meredith (2013a).

I report this exercise in appendix table C.19. Columns 1 and 2 are the OLS and 2SLS estimates for the adapted specification of equation 6 to the coattail effects framework, and columns 3 and 4, the analogous OLS and 2SLS estimates for the adapted inverse specification of equation 6.

In IV analysis, a 1p.p. increase in supporting council candidates' polling station vote share turns into a 0.44p.p. increase in the supported mayoral candidate's polling station vote share. On the other hand, the IV analysis reveals that a 1p.p. increase in the supported mayoral candidate's vote share leads to a 0.15p.p. increase in supporting council candidates' vote share. These results go hand in hand with the main spatial vote concentration's results, particularly supporting the Brokerage Hypothesis. There are two-side coattail effects, but the effect of supporting council candidates on the supported mayoral candidate is stronger.

5.2.C Heterogeneity

In the appendix, heterogeneity analyses for the Brokerage Hypothesis' test are reported. Again, I consider three dimensions that might affect the effect of allied council candidates' spatial vote concentration on the supported mayoral candidate's spatial vote concentration: municipality size, mayoral candidate incumbency status, and mayoral candidate party. For each analysis, I consider the same specifications as those of equations 6 and 7, instrumenting the dependent variable with the same *Friends-and-Neighbors*-like instrument described in section 3.2, but restricting the sample of mayoral candidacies according to the segmentation of each heterogeneity dimension.

First, table C.21 presents the municipality size heterogeneity. In each column, the sample of mayoral candidacies is restricted according to the municipalities' number of polling stations. The first column ($\#PS - Q1$) considers the sample of mayoral candidacies supported by a mayoral electoral coalition in municipalities where the number of polling stations is below the first quartile of the distribution of the municipalities' number of polling stations. The second ($\#PS - Q2$), between the first and second quartiles; the third ($\#PS - Q3$), between the second and the third quartiles; and the fourth ($\#PS - Q4$), above the third quartile. The 2SLS point estimates indicate that the marginal effect of allied council candidates' spatial vote concentration on the supported mayoral candidate's spatial vote concentration is, in general, similar across the different sizes of municipalities.

Second, table C.25 reports the mayoral incumbency status heterogeneity. The first column restricts the sample to incumbent mayoral candidacies (i.e. the mayoral candidate is the incumbent mayor or, if no candidate is the incumbent mayor, the mayoral candidate's party is the incumbent mayoral party), while the second column considers the complement (i.e. mayoral candidacies that are not incumbent mayoral candidacies). The 2SLS estimate indicates that the allied council brokerage effect is stronger when the mayoral candidacy is not the incumbent candidacy, but the marginal positive effect is statistically significant for both types of mayoral candidacy.

Finally, table C.29 shows the mayoral party heterogeneity. Each column restricts the sample to mayoral candidacies from one of the five parties that launched the most mayoral candidates in the 2020 municipality elections; i.e., MDB, PSD, PP, PSDB, and PT. The 2SLS estimates indicate that the Brokerage Hypothesis is rejected only when considering PT's mayoral candidacies. For MDB, PSD, PP, and PSDB, the results show a positive and statistically significant marginal effect of the mayoral candidate's spatial vote concentration on allied council candidates' spatial vote concentration. Interestingly, its also only when considering PT's mayoral candidacies that the inverse model estimation (in table C.31) finds a positive, strong, and statistically significant marginal effect of the mayoral candidate's spatial vote concentration on allied council candidates' spatial vote concentration. In other words, the analysis shows that in PT's mayoral candidacies, it is not allied council candidates that act as local brokers for the mayoral candidate, but rather the mayoral candidate's dominance areas that affect her allied council candidates' dominance areas. This intriguing pattern invites further exploration that is beyond the scope of this paper.

5.2.D Effective Candidates

As discussed before, a possible problem related to using the spatial vote concentration measures is that there might be mayoral and council candidates that are not competitive and get few votes. The main spatial concentration measure, the S Index, can be distorted, assuming extreme values, when a candidate has few votes (see section 2.1). Thus, I propose accessing the Brokerage Hypothesis through an alternative specification to prevent this "weak candidates" problem.

The alternative specification consists of estimating table 2's models considering only effective mayoral and city council candidates, as defined in footnote 7. The results are presented in table C.33. Point estimates are close to tables 2's main estimates, supporting the Brokerage Hypothesis: a marginal increase in allied council candidates' spatial vote concentration does increase the supported mayoral candidate's spatial vote concentration, as expected from the Brokerage Hypothesis. Again, the positive and statistically significant effect is robust to alternative measures of spatial vote concentration.

5.2.E Controlling for Mayoral Candidate's Polling Place

To strengthen the argument on the exclusion restriction validity, I propose to include in Equation 6' specification an indicator variable equal to 1 when the mayoral candidate votes at the polling place as a covariate. If the argument that candidates do not change where they vote depending on the configuration of the elections is true, then the inclusion of this indicator should not change the council candidates' vote concentration estimated effect on the supported mayoral candidate's vote concentration. In particular, a possible violation of the exclusion restriction should show up if allied council candidates vote at a particular polling place because it is where their supported mayoral candidate votes. If the inclusion of the mayoral candidate's polling place indicator as a covariate does not change the estimated effects, then it would be a piece of evidence supporting that candidates' polling places are indeed exogenously determined.

The results of this exercise are presented in the appendix table C.35. Reassuringly, point estimates are almost numerically identical to those presented from the main specification, in table 2, indicating that the inclusion of the mayoral candidate's polling place indicator as a covariate does not change council candidates' vote concentration estimated effects on the supported mayoral candidate's vote concentration, for all alternative measures of spatial vote concentration. This piece of evidence thus strengthens the argument for the validity of the instrument's exclusion restriction.

5.2.F Placebo Coalitions

As with the Spatial Dependence Hypothesis test, I also perform a placebo treatment exercise in the spirit of a permutation test for the Brokerage Hypothesis test. I consider the same 250 repetitions of the exercise performed for the Spatial Dependence Hypothesis test, where mayoral candidacies in a municipality election are taken as given and a placebo mayoral electoral coalition is constructed by randomly selecting for each party participating in the city council election a mayoral candidacy to be supported, including the possibility of supporting no candidacy. For each set of placebo mayoral electoral coalition, I estimate the models in table 2's columns 1 and 2 (i.e., the fixed-effects model and the 2SLS model). In the figure below I present the distribution of placebo-allied council candidates' vote concentration estimated effect on the supported mayoral candidate's vote concentration; Table 2's estimated effects are indicated by a vertical dotted line.

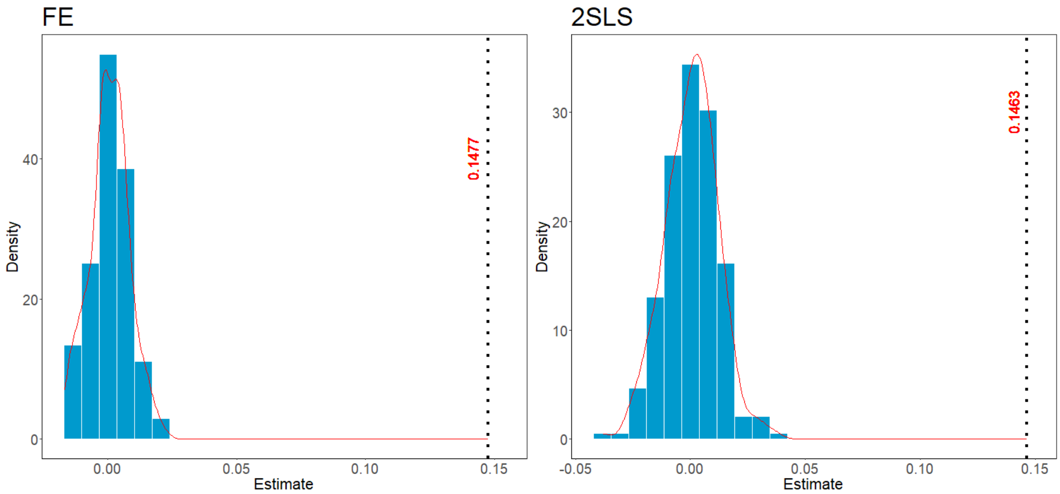

Figure 7: Placebo Coalitions - Estimates Histograms

The estimated effects considering the real coalitions, both in the complete fixed-effects and in the IV specifications, are outliers in the distributions of placebo coalitions' estimated effects. Thus, the exercise reinforces that a marginal increase in allied council candidates' spatial vote concentration indeed increases the supported mayoral candidate's spatial vote concentration, as expected from the Brokerage Hypothesis.

6 Conclusion

This research offers new insights into the dynamics of electoral alliances in local elections by analyzing their impact on candidates' spatial patterns of vote. By focusing on the 2020 Brazilian municipal elections, this study sheds light on how mayoral electoral coalitions affect the spatial vote distributions of both mayoral and allied city council candidates.

Empirical analysis supports the hypotheses that (i) electoral alliances between mayoral and city council candidates increase the spatial dependence of their vote distributions and that (ii) council candidates act as local brokers for the mayoral candidate. This increased spatial dependence is primarily due to the spatial concentration areas of supporting council candidates becoming those of the supported mayoral candidate. Thus, the Brazilian case study illustrates how electoral coalitions can affect campaign effectiveness by leveraging the localized support bases of allied candidates running for different offices.

Although this study focused on the Brazilian context, the methodological approach presented here can be applied to analyze the effects of electoral alliances or other forms of electoral support agreements on candidates' spatial vote patterns in various contexts. For example, in the United States, local elections often feature cooperation between mayoral candidates and city council members. Studying these alliances could reveal whether similar spatial voting dependencies and brokerage interplay occur in a different electoral and institutional context. Additionally, in European multiparty systems, where electoral alliances are also common, examining the impact of these alliances on spatial voting patterns could provide insights into how coalition strategies affect allied candidates' spatial patterns of vote.

Overall, this research represents a step forward in understanding the spatial dynamics of electoral alliances, providing a framework for future studies on the interplay between electoral coalition strategies and spatial patterns of vote.

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Online Appendix to "Electoral Alliances and Spatial Patterns of Vote"

Gabriel C. Caseiro

September 21, 2024

A Alternative Measures

A.1 Spatial Vote Concentration

- Horizontal Clustering (HQ):

$$\mathcal{HC}_{ilm} := V_{ilm} - V_{im} * \frac{V_{lm}}{V_m} \tag{A.9}$$

- Location Quotient (LQ):

$$\mathcal{LQ}_{ilm} := \frac{V_{ilm}}{V_{im}} / \frac{V_{lm}}{V_m} \tag{A.10}$$

- S_{Max}:

$$S_{im}^{Max} := \max_l \{S_{ilm}\} \tag{A.11}$$

- C Index:

$$C_{im} := \frac{\sum_l (\frac{V_{ilm}}{V_{im}})^2}{\sum_l (\frac{V_{lm}}{V_m})^2} \tag{A.12}$$

A.2 Spatial Vote Dependence

- Spatial Adjusted Correlation (SAC):

$$Corr_{ijm}^L := 100 * \frac{1 + corr(S_{ilm}, S_{jlm})}{2} \tag{A.13}$$

B Descriptive Statistics

B.1 Electoral Alliances and Polling Places Stats

Year	# Candidacies	Mean Candidacies by Municipality	% Candidacies with Coalition	% Municipalities with Coalition	Mean Allied Parties in Coalition
2000	15041	2.71	73.61	97.16	2.58
2004	15994	2.88	79.74	99.28	3.25
2008	15361	2.76	83.53	99.75	3.82
2012	15419	2.77	85.20	99.80	4.52
2016	16354	2.94	83.80	99.86	4.81
2020	18979	3.41	64.30	97.57	2.42

Table B.1: Mayoral Coalitions in Brazilian Municipality Elections

# Pooling Stations	N
1	213
2	489
3	416
4	413
5	364
6	364
7	301
8	255
9	224
10	205
11	162
12	164
13	154
14	130
15+	1715

Table B.2: 2020 Elections Municipalities' Number of Polling Places - Distribution

# Pooling Stations	
Mean	17
Median	8
Max	2062

Table B.3: 2020 Elections Municipalities' Number of Polling Places - Summary

B.2 Spatial Vote Concentration Stats

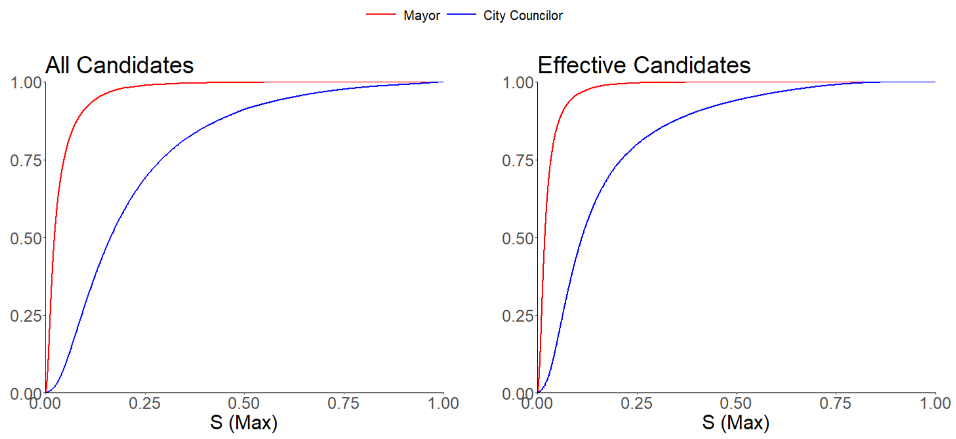

Figure B.1: Empirical CDF - S_{Max}

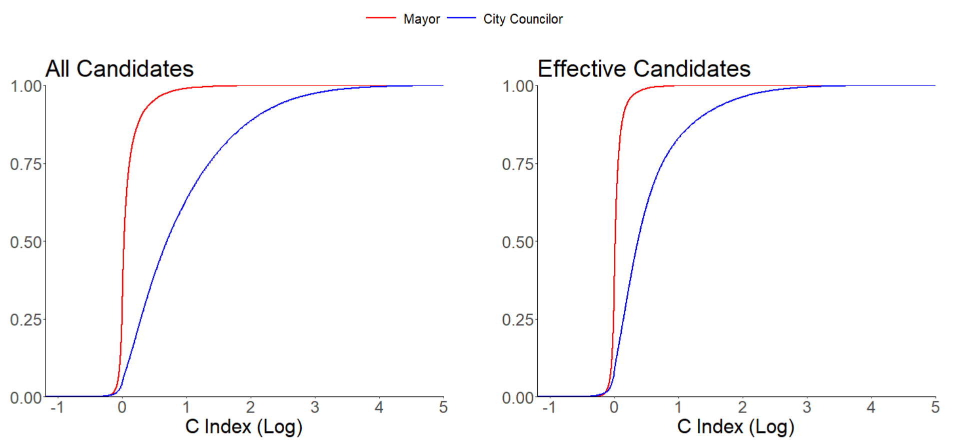

Figure B.2: Empirical CDF - C Index

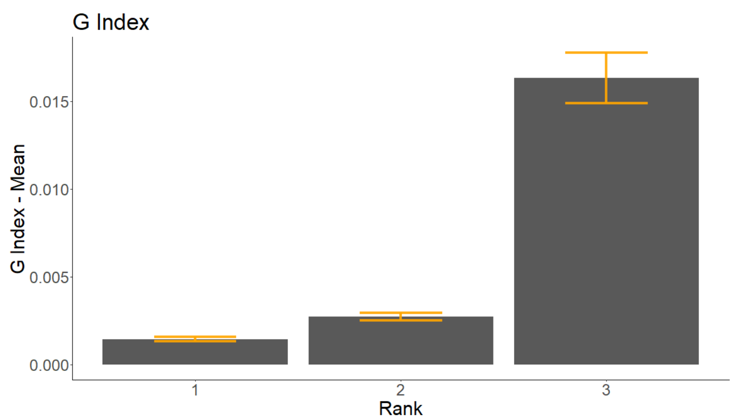

Figure B.3: Mean Concentration by Voting Rank - Mayoral Elections (Top 3)

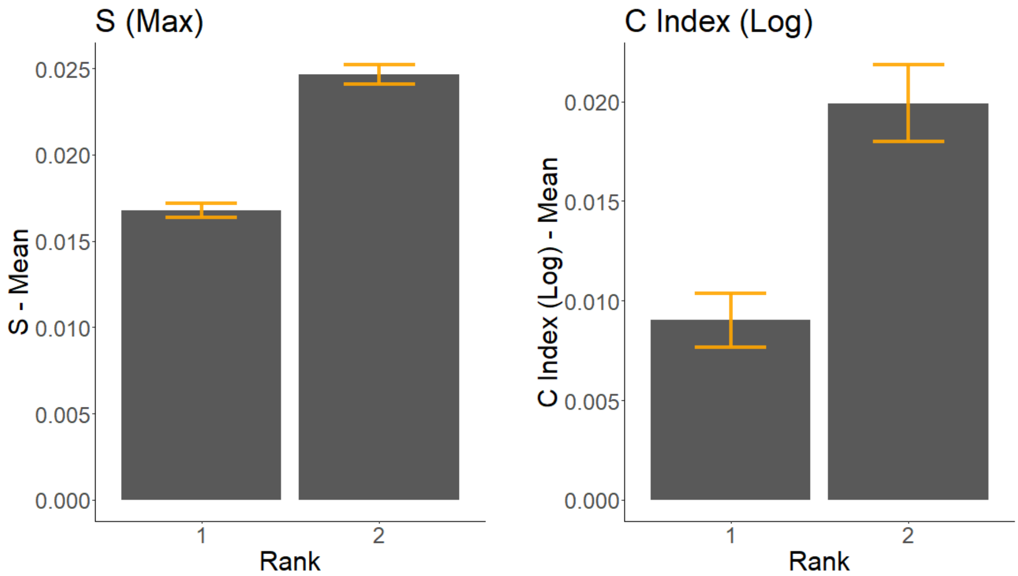

Figure B.4: Mean Concentration by Voting Rank - Mayoral Elections (S_{Max} and C Index)

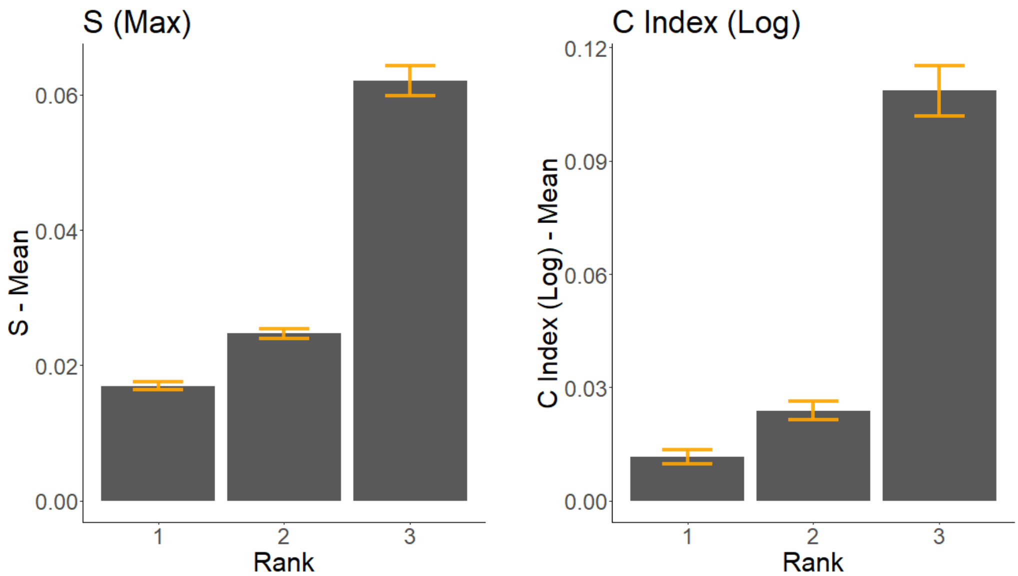

Figure B.5: Mean Concentration by Voting Rank - Mayoral Elections (Top 3 - S_{Max} and C Index)

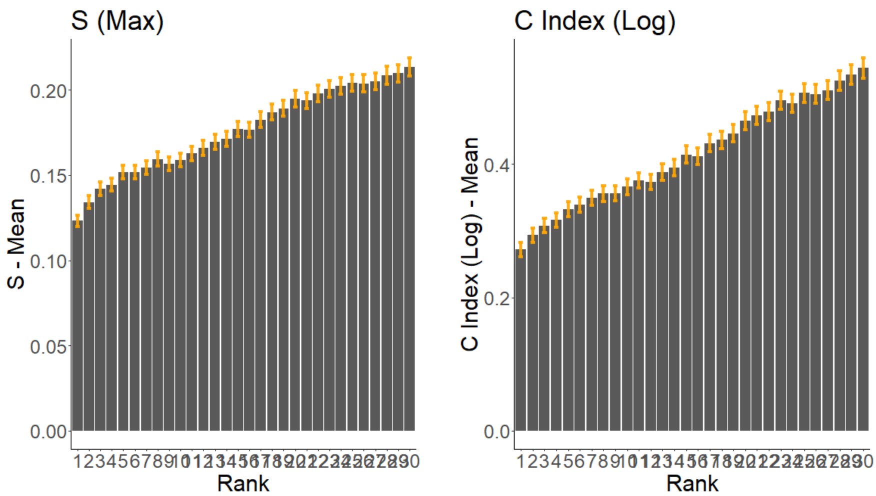

Figure B.6: Mean Concentration by Voting Rank - Council Elections (S_{Max} and C Index)

B.3 Spatial Vote Dependence Stats

	Mean	Median	SD	#
All Pairs	0.0299	-0.0034	2.2427	2504382
Excluding Mayoral Parties' CC Cand	0.0122	-0.0050	1.9451	1424901
Effective Cand	0.0052	-0.0009	2.3637	604433
Excluding Mayoral Parties' CC Cand & Effective Cand	-0.0034	-0.0016	2.0464	309970

Table B.4: SVD Index - Summary

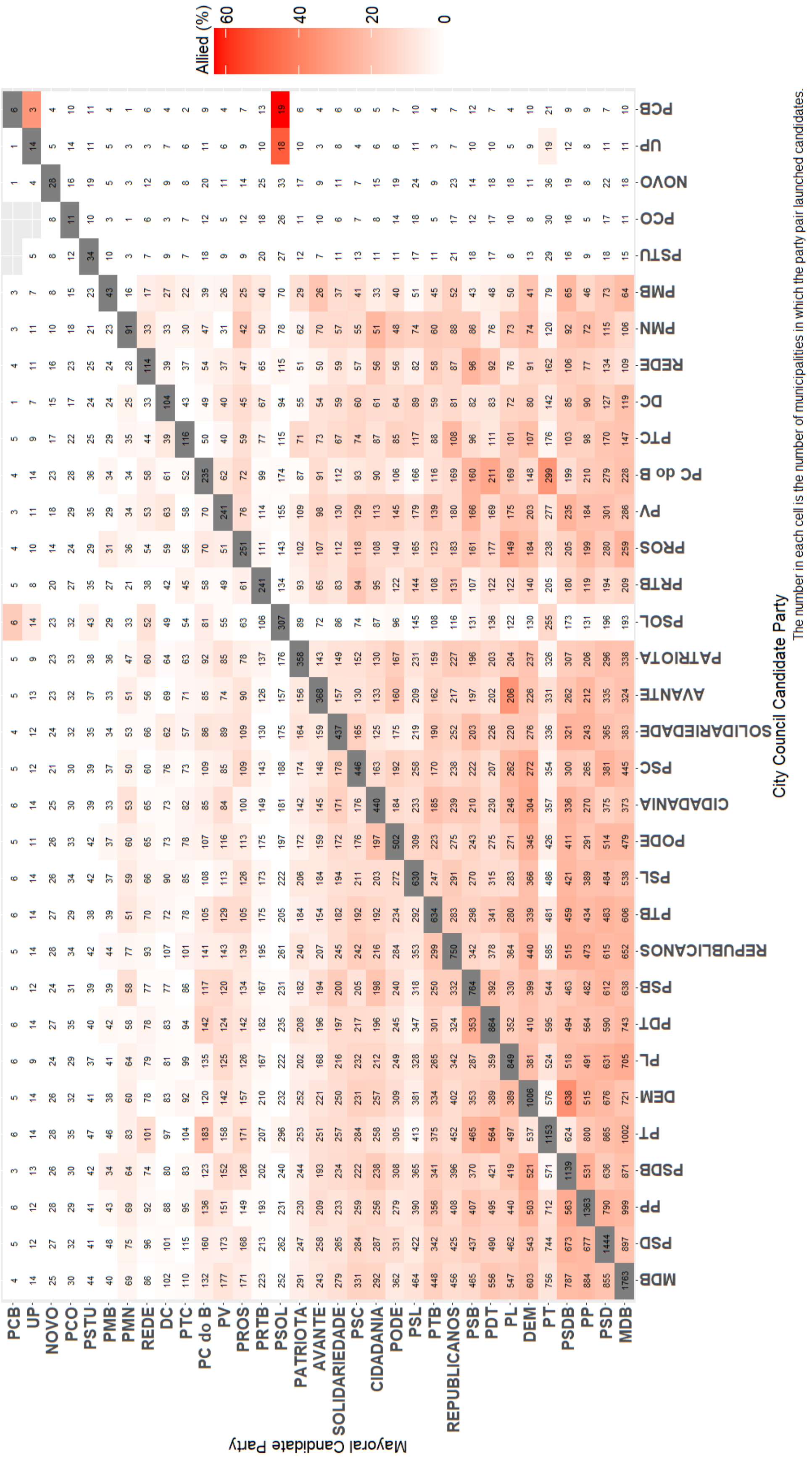

Figure B.7: 2020 Elections - Mean Alliance in Electoral Coalitions

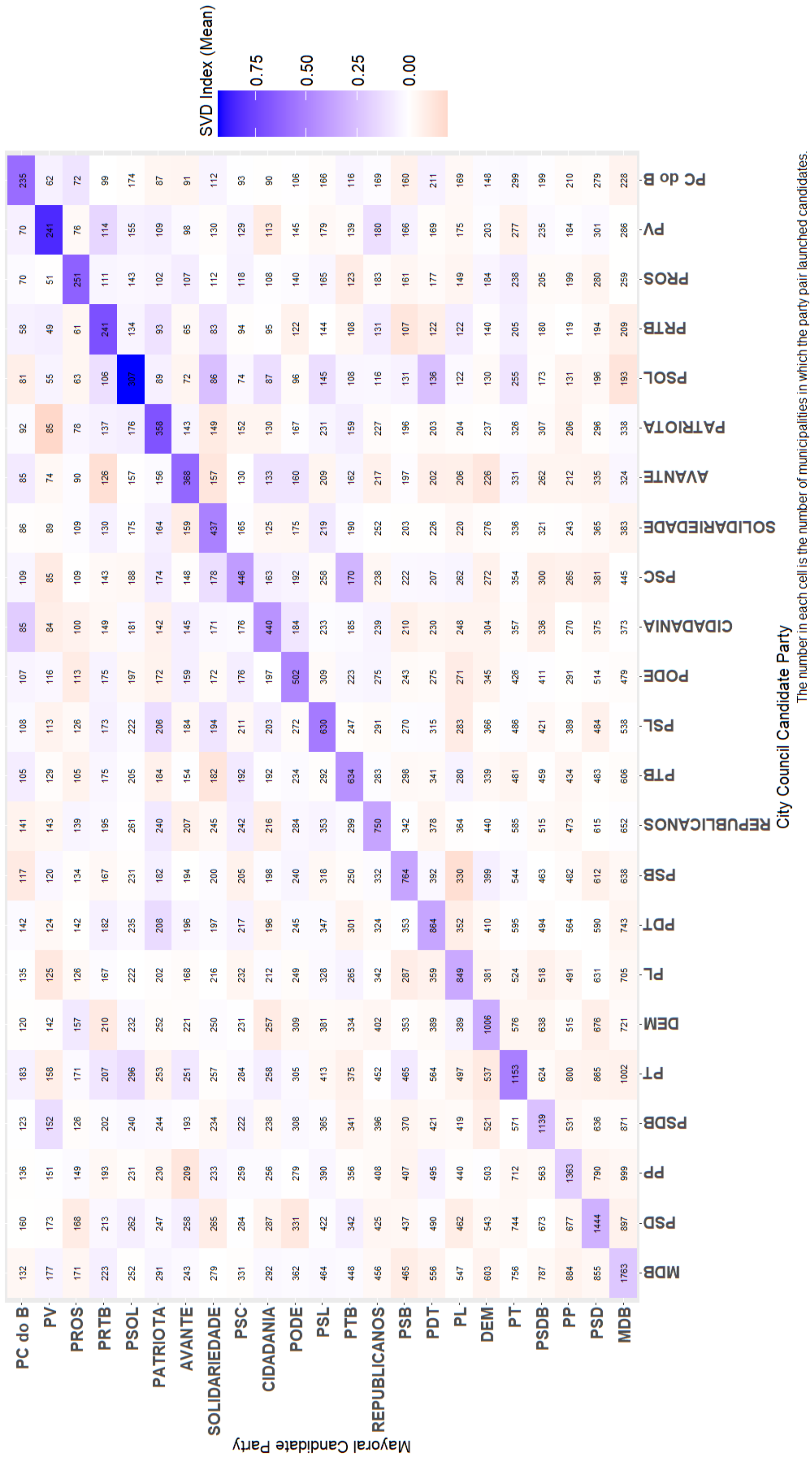

Figure B.8: Mean SVD Index - All Pairs

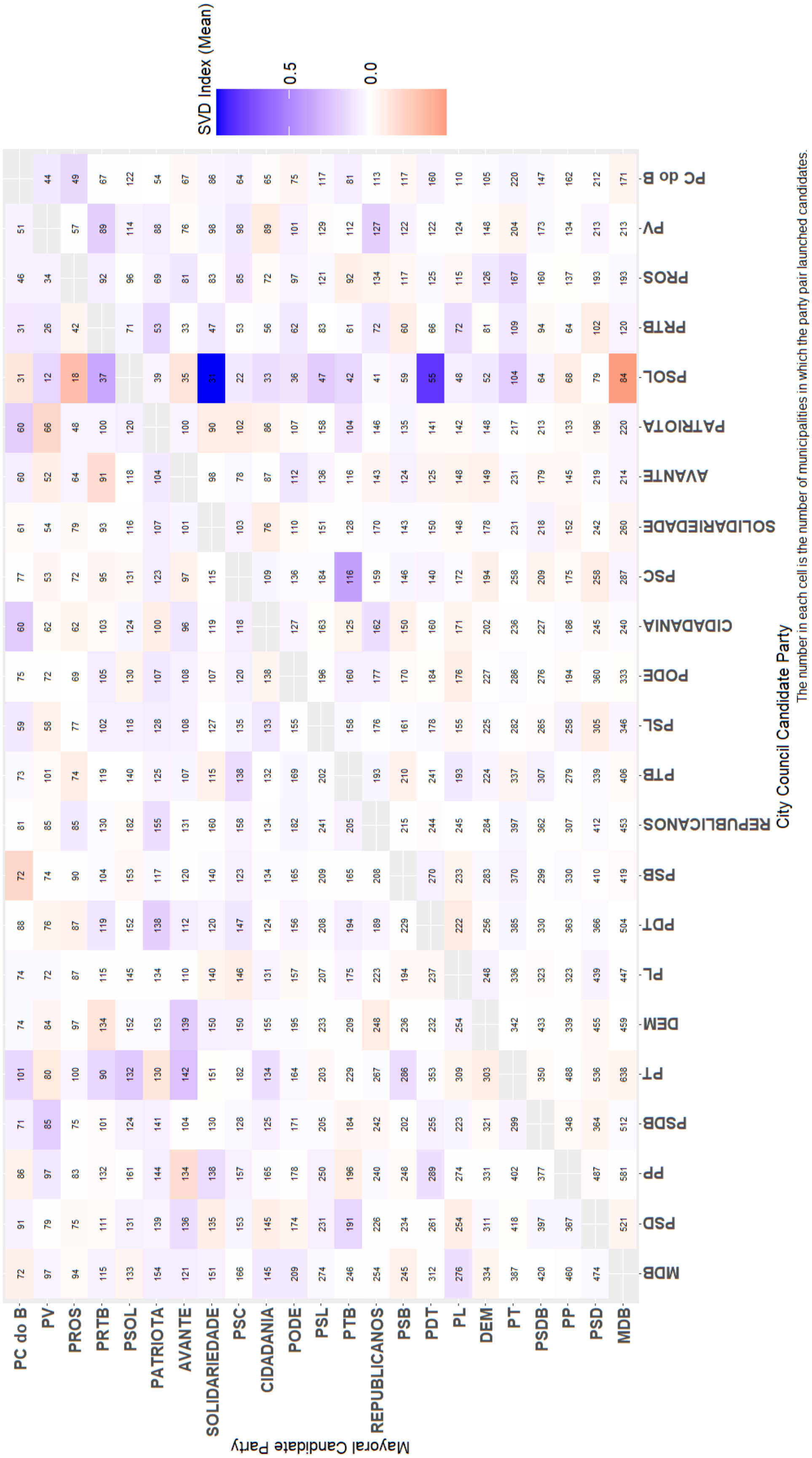

Figure B.9: Mean SVD Index - Excluding Mayoral Parties' Council Candidates

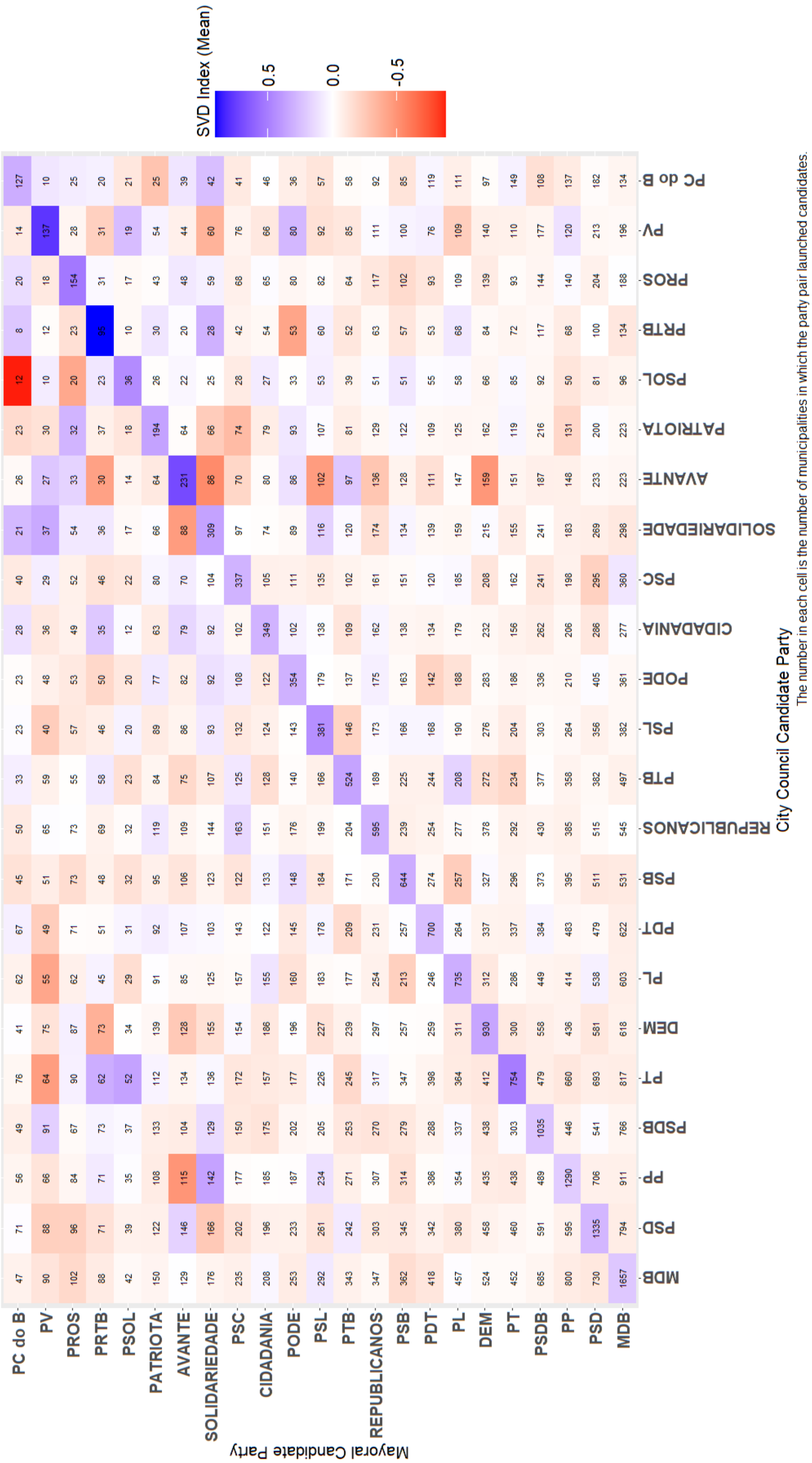

Figure B.10: Mean SVD Index - Effective Candidates

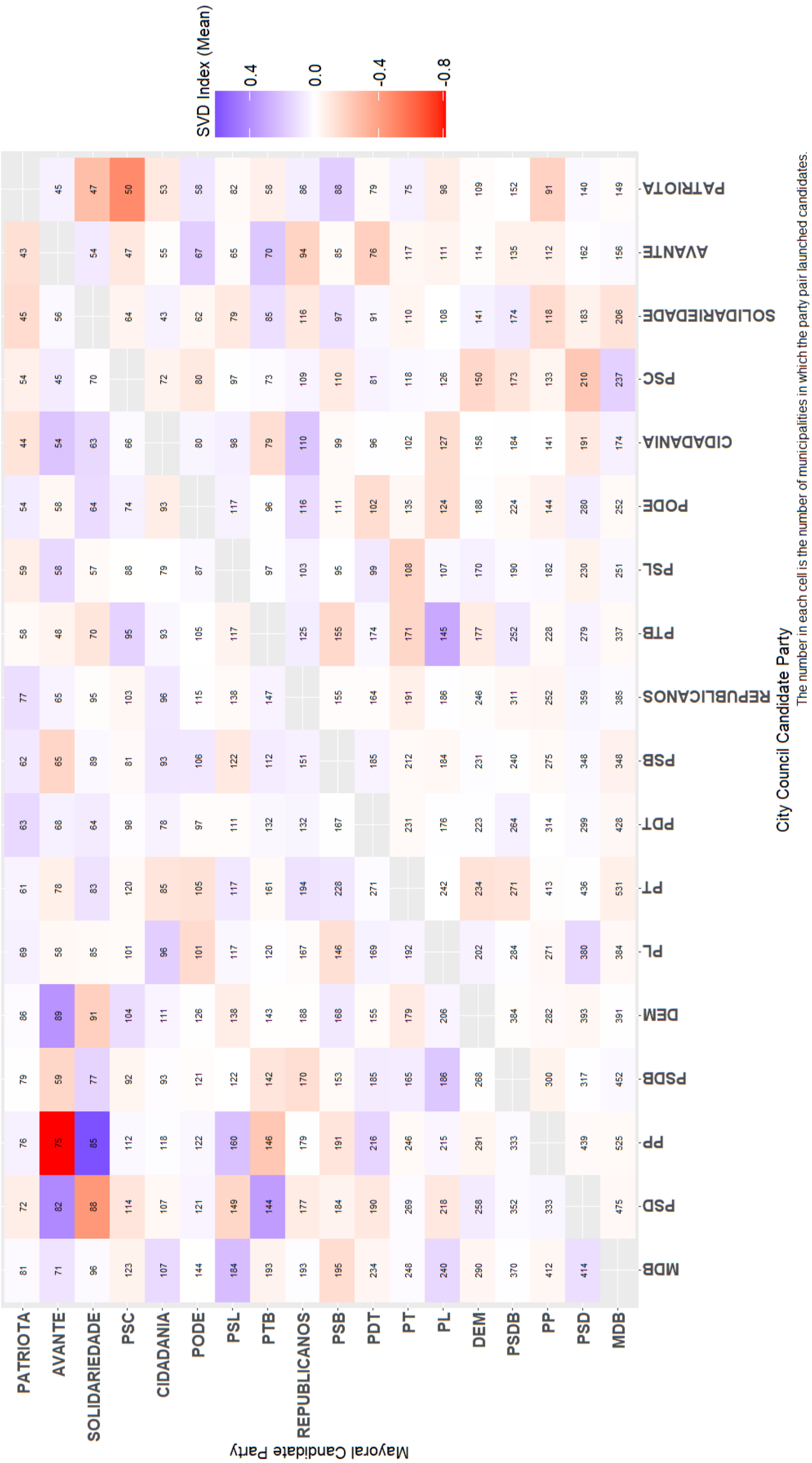

Figure B.11: Mean SVD Index - Excluding Mayoral Parties' Council Candidates and Only Effective Candidates

C Results and Extensions

C.1 Spatial Dependence Hypothesis

C.1.A Same-Party Baseline

Dependent Variable:	SVD Index	
	OLS	FE
Model:	(1)	(2)
Variables		
Same Party	0.4099***	0.5095***
	(0.0139)	(0.0152)
Fixed-effects		
Mayoral Cand. i		Yes
City Council Cand. j		Yes
Fit statistics		
Observations	2,504,382	2,504,382

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.1: Spatial Dependence Hypothesis' Test - Baseline Same Party

C.1.B Alternative Measure

Dependent Variable:	Spatial Adjusted Correlation (SAC)			
	OLS	FE		
Model:	(1)	(2)	(3)	(4)
Variables				
Allied	1.583***	2.031***	3.429***	3.537***
	(0.0813)	(0.0861)	(0.0838)	(0.0879)
Fixed-effects				
Party Pair & State		Yes		Yes
Mayoral Cand. i			Yes	Yes
City Council Cand. j			Yes	Yes
Fit statistics				
Observations	1,424,899	1,424,899	1,424,899	1,424,899

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.2: Spatial Dependence Hypothesis' Test - SAC

C.1.C IV Analysis

Consider a mayoral candidate i and a council candidate j, both from municipality m, and let P_i and P_j be their respective parties. If $M_{ms}^{(1)}$ is defined as the set of the municipalities excluding m and its border neighboring municipalities (i.e. neighboring municipalities up to the first degree, as indicated by the superscript "$^{(1)}$"), the *leave-one-out* instrument, Z_{ijms}, is given by:

$$Z_{ijms} := \frac{\left| \left\{ m' \in M_{ms}^{(1)} : \begin{array}{c} P_i \text{ and } P_j \text{ nominate candidates and are} \\ \text{in the same mayoral electoral coalition} \end{array} \right\} \right|}{\left| \{ m' \in M_{ms}^{(1)} : P_i \text{ and } P_j \text{ nominate candidates} \} \right|} \tag{C.14}$$

where the numerator is the number of municipalities, excluding m and its border neighboring municipalities, where the parties P_i and P_j are allied in a mayoral electoral coalition (which includes the cases where both parties do not launch a mayoral candidacy but support the same mayoral candidate from another party); and the denominator is the number of municipalities, excluding m and its border neighboring municipalities, where the parties P_i and P_j are participating in the elections (i.e. nominating mayoral or city council candidates).

To be a valid instrument, Z_{ijms} must be correlated with the mayoral electoral coalition indicator ($Allied_{ijms}$) and impact the spatial vote dependence of the candidates pair (SVD_{ijms}), conditional on the set of fixed-effects of Equation 4, only through the impact on the mayoral electoral coalition indicator.

The first restriction, the *relevance restriction*, is expected to be true, as the mean tendency of alliance in further municipalities should be correlated with the tendency of alliance in a particular municipality due to, for example, programmatic similarities and national issues. As for the *exclusion restriction*, which cannot be tested, it seems reasonable that, conditional on the set of fixed-effects from Equation 4, and, in particular, on the state pair of parties fixed-effect, the mean tendency of alliance in farther municipalities are affecting the spatial vote dependence only through the impact on the mayoral electoral coalition indicator.[1]

Dependent Variable:	SVD Index	
	FE	2SLS
Model:	(1)	(2)
Variables		
Allied	0.1444***	0.1693***
	(0.0073)	(0.0160)
Fixed-effects		
Party Pair & State	Yes	Yes
Mayoral Cand. i	Yes	Yes
City Council Cand. j	Yes	Yes
Fit statistics		
Observations	1,424,901	1,424,901

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.3: Spatial Dependence Hypothesis' Test - IV Analysis

[1]Nevertheless, it should be acknowledged Betz et al. (2018)'s critique on the use of spatial instruments, such as the *leave-one-out* instrument.

Dependent Variable:	Allied
Model:	(1)
Variables	
d_coli_v1	-136.2***
	(4.429)
Fixed-effects	
Party Pair & State	Yes
Mayoral Cand. i	Yes
City Council Cand. j	Yes
Fit statistics	
Observations	1,424,901

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.4: Spatial Dependence Hypothesis' Test - First Stage

C.1.D IV Analysis - Robustness

To attenuate concerns on the validity of the proposed instrument, in the appendix I also report a robustness check to the Spatial Dependence Hypothesis' *leave-one-out* instrument, defined in Equation C.14. I consider alternative definitions, changing the set of municipalities excluded from the calculation of party pairs' probability of alliance. In the primary definition, the instrument is defined over the set $M_{ms}^{(1)}$, i.e. the set of municipalities excluding m and its bordering neighbors. The robustness exercise considers the sets $M_{ms}^{(0)}$ (excluding m), $M_{ms}^{(2)}$ (excluding m, its bordering neighbors, and the bordering neighbor of its bordering neighbors; i.e. excluding m and its neighbors up to the second degree), $M_{ms}^{(3)}$ (excluding m and its neighbors up to the third degree), $M_{ms}^{(4)}$ (excluding m and its neighbors up to the fourth degree) and $M_{ms}^{(5)}$ (excluding m and its neighbors up to the fifth degree).

The robustness exercise is reported in table C.5. Considering the five different instrument definitions, the 2SLS estimates again confirm that being in the same mayoral electoral coalition increases mayoral and council candidates' spatial vote dependence. Point estimates are all positive and statistically significant, reinforcing that being in the same mayoral electoral coalition indeed increases mayoral and council candidates pair's spatial vote dependence, as stated in the Spatial Dependence Hypothesis. First-stage estimates are reported in table C.6.

Dependent Variable:	SVD Index					
	2SLS (V0)	2SLS (V1)	2SLS (V2)	2SLS (V3)	2SLS (V4)	2SLS (V5)
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
Allied	0.1542***	0.1693***	0.1618***	0.1839***	0.2442***	0.2566***
	(0.0090)	(0.0160)	(0.0302)	(0.0394)	(0.0564)	(0.0726)
Fixed-effects						
Party Pair & State	Yes	Yes	Yes	Yes	Yes	Yes
Mayoral Cand. i	Yes	Yes	Yes	Yes	Yes	Yes
City Council Cand. j	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	1,424,901	1,424,901	1,424,901	1,424,901	1,424,901	1,424,901

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.5: Spatial Dependence Hypothesis' Test - Instrument Robustness

Dependent Variable:	Allied					
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
d_coli_v0	-525.7***					
	(11.93)					
d_coli_v1		-136.2***				
		(4.429)				
d_coli_v2			-63.03***			
			(1.697)			
d_coli_v3				-37.31***		
				(1.030)		
d_coli_v4					-24.10***	
					(0.7916)	
d_coli_v5						-16.69***
						(0.6739)
Fixed-effects						
Party Pair & State	Yes	Yes	Yes	Yes	Yes	Yes
Mayoral Cand. i	Yes	Yes	Yes	Yes	Yes	Yes
City Council Cand. j	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	1,424,901	1,424,901	1,424,901	1,424,901	1,424,901	1,424,901

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.6: Spatial Dependence Hypothesis' Test - Instrument Robustness (First Stage)

C.1.E Heterogeneity

Dependent Variables:	SVD Index	Spatial Adjusted Correlation (SAC)
Model:	(1)	(2)
Variables		
Allied	0.4158***	6.225***
	(0.0781)	(0.7546)
Allied*Q2_PS	-0.2045**	-1.368*
	(0.0837)	(0.8160)
Allied*Q3_PS	-0.2253***	-1.948**
	(0.0798)	(0.7766)
Allied*Q4_PS	-0.3239***	-3.437***
	(0.0780)	(0.7593)
Fixed-effects		
Party Pair & State	Yes	Yes
Mayoral Cand. i	Yes	Yes
City Council Cand. j	Yes	Yes
Fit statistics		
Observations	1,424,901	1,424,899

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.7: Spatial Dependence Hypothesis - Municipality Size Heterogeneity

Dependent Variables:	SVD Index	Spatial Adjusted Correlation (SAC)
Model:	(1)	(2)
Variables		
Allied	0.1618***	3.635***
	(0.0106)	(0.1087)
Allied*Incumbent Mayor	-0.0510***	-0.2744
	(0.0195)	(0.2076)
Fixed-effects		
Party Pair & State	Yes	Yes
Mayoral Cand. i	Yes	Yes
City Council Cand. j	Yes	Yes
Fit statistics		
Observations	1,411,225	1,411,223

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.8: Spatial Dependence Hypothesis - Mayoral Incumbency Heterogeneity

Dependent Variables:	SVD Index	Spatial Adjusted Correlation (SAC)
Model:	(1)	(2)
Variables		
Allied	0.1581***	3.620***
	(0.0114)	(0.1186)
Allied*MDB	-0.0024	0.1458
	(0.0287)	(0.3166)
Allied*PSD	-0.0552**	-0.5512*
	(0.0262)	(0.3256)
Allied*PP	-0.0184	-0.4302
	(0.0307)	(0.3486)
Allied*PSDB	-0.0524**	0.0505
	(0.0232)	(0.3358)
Allied*PT	-0.0117	-0.1989
	(0.0468)	(0.4493)
Fixed-effects		
Party Pair & State	Yes	Yes
Mayoral Cand. i	Yes	Yes
City Council Cand. j	Yes	Yes
Fit statistics		
Observations	1,424,901	1,424,899

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.9: Spatial Dependence Hypothesis - Mayoral Party Heterogeneity

C.1.F Party Aggregation and Effective Candidates

Dependent Variable:	SVD Index				
	OLS		FE		2SLS
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Allied	0.1610***	0.1855***	0.2460***	0.2487***	0.2685***
	(0.0103)	(0.0123)	(0.0146)	(0.0152)	(0.0257)
Fixed-effects					
Party Pair & State		Yes		Yes	Yes
Mayoral Cand. i			Yes	Yes	Yes
City Council Cand. j			Yes	Yes	Yes
Fit statistics					
Observations	98,622	98,622	98,622	98,622	98,622

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.10: Spatial Dependence Hypothesis' Test - Council Parties

Dependent Variable:	SVD Index				
	OLS		FE		2SLS
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Allied	0.1495***	0.1628***	0.2230***	0.2363***	0.2447***
	(0.0087)	(0.0098)	(0.0129)	(0.0150)	(0.0275)
Fixed-effects					
Party Pair & State		Yes		Yes	Yes
Mayoral Cand. i			Yes	Yes	Yes
City Council Cand. j			Yes	Yes	Yes
Fit statistics					
Observations	309,970	309,970	309,970	309,970	309,970

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.11: Spatial Dependence Hypothesis' Test - Effective Candidates

C.1.G Donations

Dependent Variable:	SVD Index				
	OLS		FE		2SLS
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Allied	0.0695***	0.0923***	0.1208***	0.1304***	0.1719***
	(0.0055)	(0.0065)	(0.0077)	(0.0087)	(0.0249)
Donation (i to j)	0.0159**	0.0124	0.0345***	0.0343**	-0.0064
	(0.0080)	(0.0086)	(0.0132)	(0.0140)	(0.0263)
Fixed-effects					
Party Pair & State		Yes		Yes	Yes
Mayoral Cand. i			Yes	Yes	Yes
City Council Cand. j			Yes	Yes	Yes
Fit statistics					
Observations	1,424,901	1,424,901	1,424,901	1,424,901	1,424,901

Clustered (Mayoral Cand. i & City Council Cand. j) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.12: Spatial Dependence Hypothesis' Test - Inter-Candidates Donations

C.2 Brokerage Hypothesis

C.2.A Same-Party Baseline

Dependent Variables:	M Cand. S		M Cand. HC		M Cand. LQ	
	OLS	2SLS	OLS	2SLS	OLS	2SLS
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
Same Party CC Cand. S	0.2609***	0.2849***				
	(0.0094)	(0.0119)				
Same Party CC Cand. HC			0.5271***	0.6298***		
			(0.0349)	(0.0199)		
Same Party CC Cand. LQ					0.2478***	0.2651***
					(0.0128)	(0.0092)
Fixed-effects						
Polling Place	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	239,695	239,695	239,695	239,695	239,695	239,695

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.13: Brokerage Hypothesis' Test - Baseline Same Party

C.2.B 2SLS First Stage

Dependent Variables:	Allied CC Cand. S	Allied CC Cand. HC	Allied CC Cand. LQ
Model:	(1)	(2)	(3)
Variables			
Allied CC Cand. voting at PS (%)	0.0020***	2.171***	0.0352***
	(5.78×10^{-5})	(0.0600)	(0.0013)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.14: Brokerage Hypothesis' Test - First Stage

Dependent Variables:	M Cand. S	M Cand. HC	M Cand. LQ
Model:	(1)	(2)	(3)
Variables			
M Cand. voting at PS	0.0112***	49.52***	0.2046***
	(0.0005)	(2.156)	(0.0105)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.15: Brokerage Hypothesis' Test - "Inverse" First Stage

Dependent Variables:	Same Party CC Cand. S	Same Party CC Cand. HC	Same Party CC Cand. LQ
Model:	(1)	(2)	(3)
Variables			
Same Party CC Cand. voting at PS (%)	0.0017***	2.849***	0.0307***
	(4.88×10^{-5})	(0.0710)	(0.0008)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	239,695	239,695	239,695

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.16: Brokerage Hypothesis' Test - Baseline First Stage

C.2.C Standardized Variables

Dependent Variables:	M Cand. S	M Cand. HC	M Cand. LQ
		2SLS	
Model:	(1)	(2)	(3)
Variables			
Allied CC Cand. S	0.3205***		
	(0.0216)		
Allied CC Cand. HC		0.4349***	
		(0.0189)	
Allied CC Cand. LQ			0.2767***
			(0.0140)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.17: Brokerage Hypothesis' Test - Standardized

Dependent Variables:	Allied CC Cand. S	Allied CC Cand. HC	Allied CC Cand. LQ
		2SLS	
Model:	(1)	(2)	(3)
Variables			
M Cand. S	0.0840***		
	(0.0285)		
M Cand. HC		0.1299***	
		(0.0225)	
M Cand. LQ			0.1100***
			(0.0215)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.18: Brokerage Hypothesis' Test - "Inverse" Standardized

C.2.D Coattail Effects Framework

Dependent Variables:	M Cand. Vote Share (%)		Allied Vote Share (%)	
	OLS	2SLS	OLS	2SLS
Model:	(1)	(2)	(3)	(4)
Variables				
Allied Vote Share (%)	0.7534***	0.4400***		
	(0.0201)	(0.0162)		
M Cand. Vote Share (%)			0.5070***	0.1463***
			(0.0132)	(0.0189)
Fixed-effects				
Polling Place	Yes	Yes	Yes	Yes
Fit statistics				
Observations	247,792	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.19: Brokerage Hypothesis - Coattails Effects Framework

Dependent Variables:	Allied Vote Share (%)	M Cand. Vote Share (%)
Model:	(1)	(2)
Variables		
Allied CC Cand. voting at PS (%)	0.2484***	
	(0.0065)	
M Cand. voting at PS		3.874***
		(0.1389)
Fixed-effects		
Polling Place	Yes	Yes
Fit statistics		
Observations	247,792	247,792

Clustered (Municipality) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.20: Brokerage Hypothesis - Coattails Effects Framework First Stage

C.2.E Heterogeneity

Dependent Variable:	M Cand. S			
	#PS - Q1	#PS - Q2	#PS - Q3	#PS - Q4
Model:	(1)	(2)	(3)	(4)
Variables				
Allied CC Cand. S	0.1859***	0.1127***	0.1561***	0.1500***
	(0.0485)	(0.0221)	(0.0163)	(0.0138)
Fixed-effects				
Polling Place	Yes	Yes	Yes	Yes
Fit statistics				
Observations	3,167	12,852	31,861	199,912

Clustered (Municipality) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.21: Brokerage Hypothesis - Municipality Size Heterogeneity

Dependent Variable:	Allied CC Cand. S			
	#PS - Q1	#PS - Q2	#PS - Q3	#PS - Q4
Model:	(1)	(2)	(3)	(4)
Variables				
Allied CC Cand. voting at PS (%)	0.0022***	0.0023***	0.0021***	0.0017***
	(0.0004)	(0.0001)	(9.8×10^{-5})	(6.45×10^{-5})
Fixed-effects				
Polling Place	Yes	Yes	Yes	Yes
Fit statistics				
Observations	3,167	12,852	31,861	199,912

Clustered (Municipality) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.22: Brokerage Hypothesis - Municipality Size Heterogeneity First Stage

Dependent Variable:	Allied CC Cand. S			
	#PS - Q1	#PS - Q2	#PS - Q3	#PS - Q4
Model:	(1)	(2)	(3)	(4)
Variables				
M Cand. S	0.5822***	-0.2561	0.2425**	0.2415***
	(0.2021)	(0.2266)	(0.1204)	(0.0621)
Fixed-effects				
Polling Place	Yes	Yes	Yes	Yes
Fit statistics				
Observations	3,167	12,852	31,861	199,912

Clustered (Municipality) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.23: Brokerage Hypothesis - Municipality Size Heterogeneity “Inverse”

Dependent Variable:	M Cand. S			
	#PS - Q1	#PS - Q2	#PS - Q3	#PS - Q4
Model:	(1)	(2)	(3)	(4)
Variables				
M Cand. voting at PS	0.0183***	0.0127***	0.0122***	0.0095***
	(0.0040)	(0.0016)	(0.0010)	(0.0005)
Fixed-effects				
Polling Place	Yes	Yes	Yes	Yes
Fit statistics				
Observations	3,167	12,852	31,861	199,912

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.24: Brokerage Hypothesis - Municipality Size Heterogeneity "Inverse" First Stage

Dependent Variable:	M Cand. S	
	Incumbent Mayor (Cand|Part)	Opponents
Model:	(1)	(2)
Variables		
Allied CC Cand. S	0.0691***	0.1280***
	(0.0183)	(0.0101)
Fit statistics		
Observations	69,487	176,884

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.25: Brokerage Hypothesis - Mayoral Incumbency Heterogeneity

Dependent Variable:	Allied CC Cand. S	
	Incumbent Mayor (Cand|Part)	Opponents
Model:	(1)	(2)
Variables		
Allied CC Cand. voting at PS (%)	0.0003***	0.0006***
	(1.79×10^{-5})	(2.02×10^{-5})
Fit statistics		
Observations	69,487	176,884

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.26: Brokerage Hypothesis - Mayoral Incumbency Heterogeneity First Stage

Dependent Variable:	Allied CC Cand. S	
	Incumbent Mayor (Cand|Part)	Opponents
Model:	(1)	(2)
Variables		
M Cand. S	-0.4294**	0.2157**
	(0.1817)	(0.0875)
Fit statistics		
Observations	69,487	176,884

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.27: Brokerage Hypothesis - Mayoral Incumbency Heterogeneity “Inverse”

Dependent Variable:	M Cand. S	
	Incumbent Mayor (Cand|Part)	Opponents
Model:	(1)	(2)
Variables		
M Cand. voting at PS	0.0042***	0.0074***
	(0.0003)	(0.0004)
Fit statistics		
Observations	69,487	176,884

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.28: Brokerage Hypothesis - Mayoral Incumbency Heterogeneity “Inverse” First Stage

Dependent Variable:	M Cand. S				
	MDB	PSD	PP	PSDB	PT
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Allied CC Cand. S	0.1141***	0.0837***	0.1042***	0.1793***	0.0304
	(0.0280)	(0.0254)	(0.0260)	(0.0392)	(0.0313)
Fit statistics					
Observations	23,136	21,313	18,372	19,501	16,774

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.29: Brokerage Hypothesis - Mayoral Party Heterogeneity

Dependent Variable:	Allied CC Cand. S				
	MDB	PSD	PP	PSDB	PT
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Allied CC Cand. voting at PS (%)	0.0004***	0.0005***	0.0004***	0.0003***	0.0007***
	(3.96×10^{-5})	(3.47×10^{-5})	(4.26×10^{-5})	(3.12×10^{-5})	(6.06×10^{-5})
Fit statistics					
Observations	23,136	21,313	18,372	19,501	16,774

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.30: Brokerage Hypothesis - Mayoral Party Heterogeneity First Stage

Dependent Variable:	Allied CC Cand. S				
	MDB	PSD	PP	PSDB	PT
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
M Cand. S	-0.3518	0.1082	0.2733	-0.4888	0.9077***
	(0.2658)	(0.3171)	(0.3281)	(0.4783)	(0.3341)
Fit statistics					
Observations	23,136	21,313	18,372	19,501	16,774

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.31: Brokerage Hypothesis - Mayoral Party Heterogeneity “Inverse”

Dependent Variable:	M Cand. S				
	MDB	PSD	PP	PSDB	PT
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
M Cand. voting at PS	0.0061***	0.0040***	0.0052***	0.0031***	0.0067***
	(0.0008)	(0.0007)	(0.0006)	(0.0008)	(0.0012)
Fit statistics					
Observations	23,136	21,313	18,372	19,501	16,774

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.32: Brokerage Hypothesis - Mayoral Party Heterogeneity “Inverse” First Stage

C.2.F Effective Candidates

Dependent Variables:	M Cand. S		M Cand. HC		M Cand. LQ	
	OLS	2SLS	OLS	2SLS	OLS	2SLS
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
s_EFCAND	0.1270***	0.1154***				
	(0.0065)	(0.0072)				
hc_EFCAND			0.2664***	0.4076***		
			(0.0319)	(0.0180)		
r_EFCAND					0.0792***	0.1003***
					(0.0129)	(0.0049)
Fixed-effects						
Polling Place	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	201,851	201,851	201,851	201,851	201,851	201,851

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.33: Brokerage Hypothesis' Test - Effective Candidates

Dependent Variables:	s_EFCAND	hc_EFCAND	r_EFCAND
Model:	(1)	(2)	(3)
Variables			
D_lv_EFCAND	0.0016***	1.898***	0.0301***
	(4.45×10^{-5})	(0.0492)	(0.0013)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	201,851	201,851	201,851

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.34: Brokerage Hypothesis' Test - Effective Candidates (First Stage)

C.2.G Controlling for Mayoral Candidate's Polling Place

Dependent Variables:	M Cand. S		M Cand. HC		M Cand. LQ	
	OLS	2SLS	OLS	2SLS	OLS	2SLS
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
Allied CC Cand. S	0.1466***	0.1463***				
	(0.0076)	(0.0096)				
M Cand. voting at PS	0.0109***	0.0109***	47.94***	46.72***	0.2005***	0.1984***
	(0.0005)	(0.0005)	(2.100)	(2.054)	(0.0104)	(0.0104)
Allied CC Cand. HC			0.2671***	0.4730***		
			(0.0318)	(0.0202)		
Allied CC Cand. LQ					0.0900***	0.1388***
					(0.0219)	(0.0070)
Fixed-effects						
Polling Place	Yes	Yes	Yes	Yes	Yes	Yes
Fit statistics						
Observations	247,792	247,792	247,792	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.35: Brokerage Hypothesis' Test - Mayoral Candidate's Polling Place Indicator

Dependent Variables:	Allied CC Cand. S	Allied CC Cand. HC	Allied CC Cand. LQ
Model:	(1)	(2)	(3)
Variables			
Allied CC Cand. voting at PS (%)	0.0020***	2.171***	0.0352***
	(5.78×10^{-5})	(0.0600)	(0.0013)
M Cand. voting at PS	0.0021***	5.908***	0.0448***
	(0.0007)	(1.070)	(0.0098)
Fixed-effects			
Polling Place	Yes	Yes	Yes
Fit statistics			
Observations	247,792	247,792	247,792

Clustered (Municipality) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table C.36: Brokerage Hypothesis' Test - Mayoral Candidate's Polling Place Indicator (First Stage)